Mathematical Tools for the Masses

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Take home messages:

* Developing mathematical tools for *mathematical modeling* should be the most important "outcome" of the mathematics taught to students as part of a public K-12 education.

* Developing modeling skills can best be achieved by emphasizing the *breadth* of the mathematics that students see over the depth of what they see.

* Most of our "customers" are not destined for STEM careers.

Mathematics K-11 curriculum should reflect this.

* More attention should be paid to making students aware of how mathematics affects their lives, in particular mathematics's role in the development of new technologies.

Let me try to construct a *mathematical model* for American society's stake in K-12 mathematics education.

Modeling Mathematics Education in Public Schools:

~ Who are the stake holders?

K-12 students

American society

American businesses

Parents of students

Politicians

Test creators

Textbook providers

The mathematics community

Mathematics teachers

~ What variables do we have control over?

* Mathematical content

* Quality of teachers who deliver mathematics in classrooms

* The style that content is delivered with

(teaching using contexts; group methods; assessment)

~Mathematical content of K -12

As someone trained as a researcher in mathematics perhaps this is the area I am most prepared to comment about.

New elementary mathematical content and applications are *constantly* being generated.

Students should see some of these new elementary mathematical tools!

~ What are our goals? What are we "optimizing" for?

I think the importance of the content we choose hasn't been properly debated.

Tools kids learn to use around the house:

Hammer

Screw driver

Saw

We know about these tools and how to use them often without formal instruction.

What are the equivalent basic tools of the mathematician?

Tools in service of themes rather than techniques

Often curriculum has been organized in terms of

Techniques

adding fractions

working with decimals

solving linear equations

adding algebraic expressions

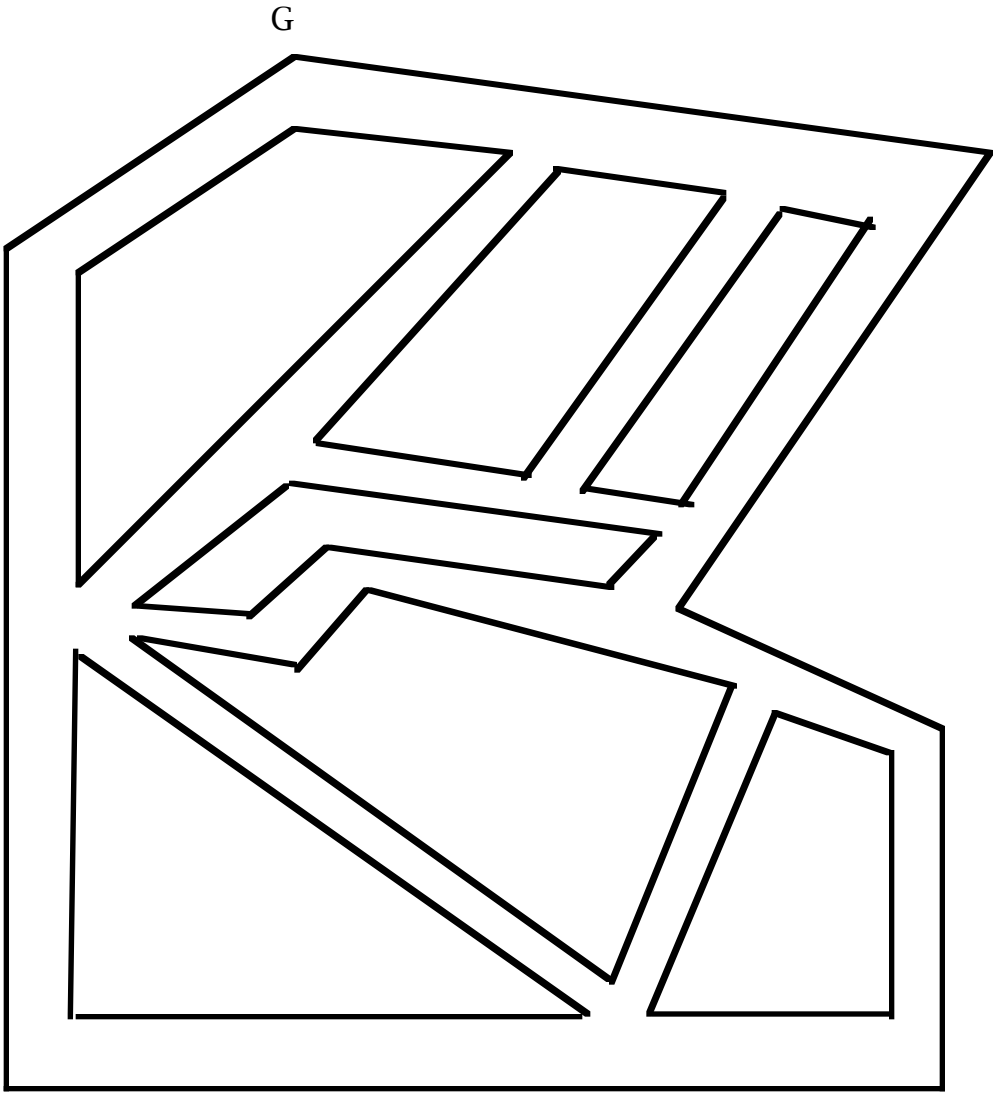
solving quadratic equations

trigonometry

Themes:

1. Optimization
2. Growth and Change
3. Information (Data)
4. Fairness and Equity
5. Risk
6. Shape and Space
7. Pattern and Symmetry
8. Order and Disorder
9. Reconstruction (from partial information)
10. Conflict and Cooperation
11. Unintuitive behavior

Example: Find an efficient pothole inspection route starting at G (Garage):



What can mathematicians, especially research mathematicians, contribute to mathematics education?

My answer:

* Elementary mathematical tools which give Americans ways to get insight into the world using mathematical ideas

* Examples that laymen can practice which show meaningful applications so that mathematics's meaning and value is clearer

We can find the future Ramanujans and Thurstons without losing everyone else!

I am not opposed to equations, but single equations (other than perhaps differential and partial differential equations) are surprisingly unuseful in the ability to give insight into new situations that arise in everyday life.

What I don't like about the CCSS-M:

Too much symbol manipulation (algebra) and not enough modeling and geometry

Practice 4 - Model with Mathematics

is much to my liking.

However, since almost no illustrations of what is meant are given, it is reasonable to assume that issues involving the high stakes tests will overwhelm what is done by way of modeling.

We tend to emphasize what effects high stakes tests might have on students but they also affect what teachers can reasonably do.

Some extended examples:

School choice: Gale-Shapley

Fair rules for the legislative bodies of the European Union (which involves mathematics similar to our Electoral College) and similar issues for the U.S.

(Don't imitate Singapore by way of illustrating "internationalism" but show some of the issues involved in the amazing experiment called the European Union.)

3. ... Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

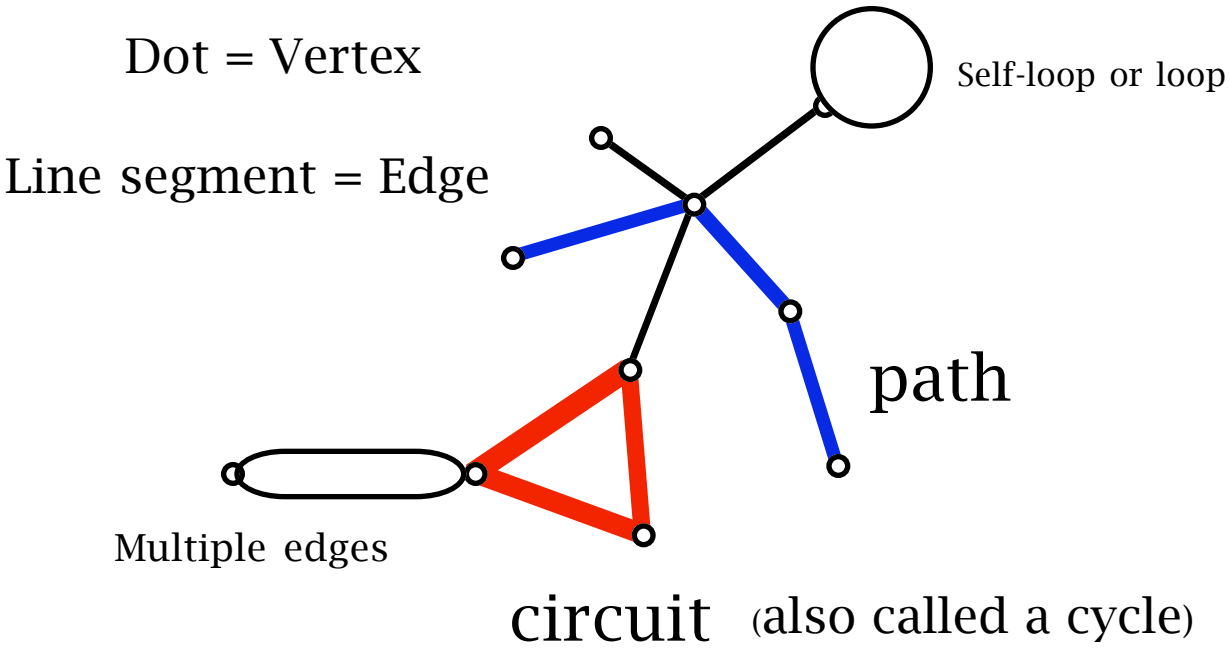
In my opinion having items like this as part of a "national curriculum" is

irrational.

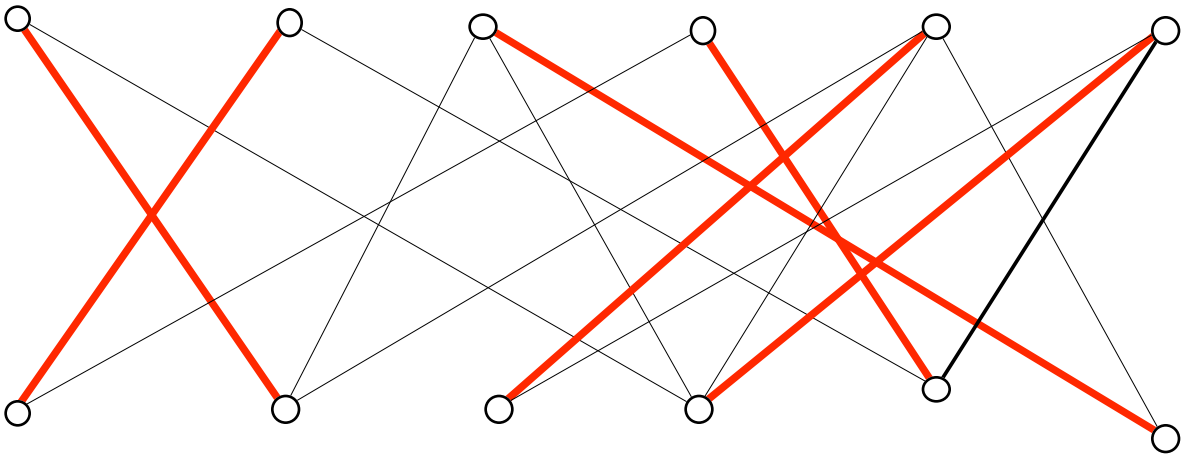
What equivalent of the hammer and saw does this "skill" serve for students?

The single most important example of a tool explicitly left out?

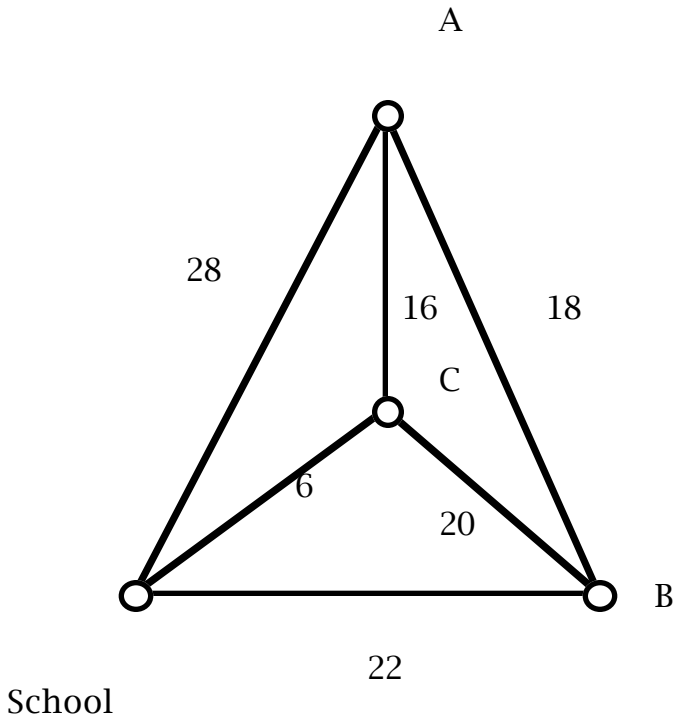
Graph:



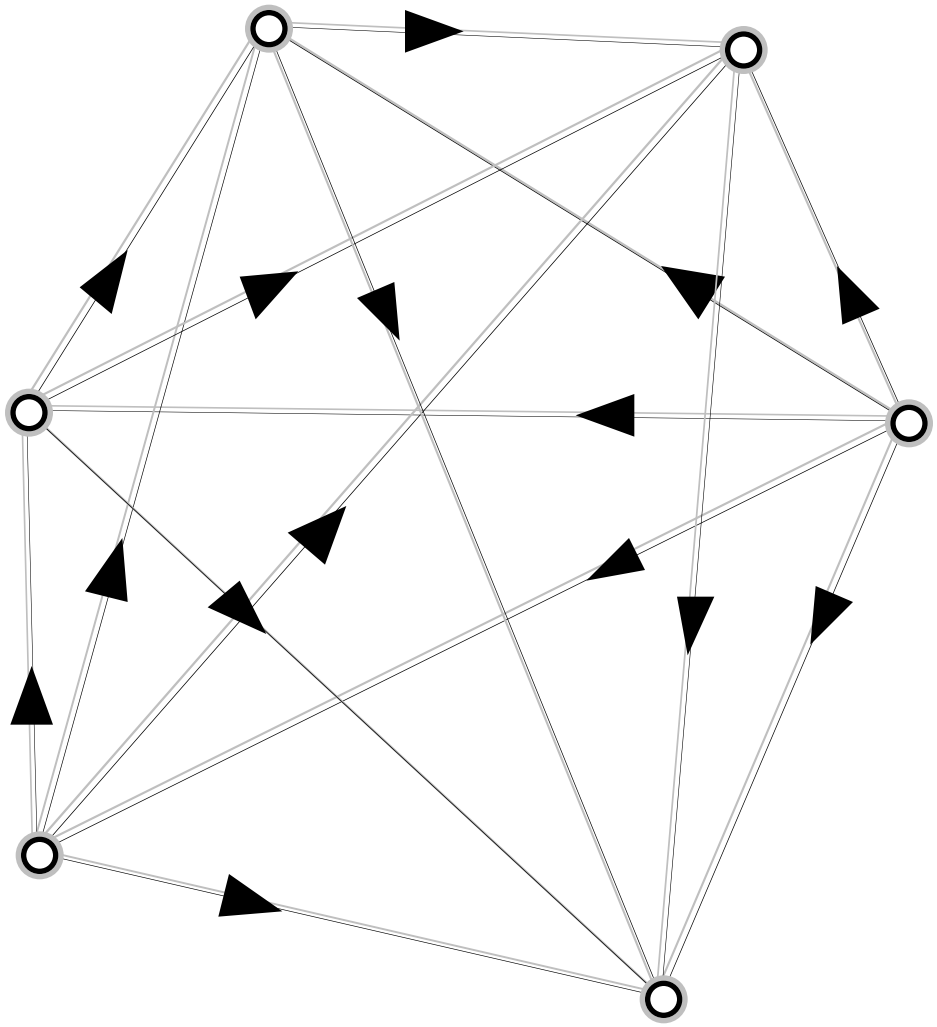
Show a graph to assign workers to jobs.



A diagram showing times to pick up students at designated bus stops to take them to summer camp at a school:



A tournament digraph:



Second most important missing topic:

Recursion equations:

Recursion or difference equations are a very natural way to model growth.

$A(t+1) = A(t) + iA(t) - M(\text{monthly regular payment}) - S(\text{Special payment})$

Why is understanding the meaning of this type of equation

$$B(n+1) = B(n) + iB(n) -.02B(n)$$

which models the unpaid balance on a credit card, where each month the minimal 2% of the remaining balance is paid, less important than this one:

$$x^2 + 5x - 6 = 0 \ ?$$

If we choose better content for grades K-12 illustrating the nature and applicability of mathematics, delivered with contexts and examples, not only will American have a better view of mathematics and mathematicians, we would in all likelihood get more Americans to follow STEM careers than we do now.

Take home messages, again:

* Developing mathematical tools for mathematical modeling should be the most important "outcome" from the mathematics students are exposed to in a public K-12 education.

* Developing modeling skills can best be achieved by emphasizing the *breadth* of the mathematics that pre-college students see over the depth of what they see.

* Most of our "customers" are not destined for STEM careers.

Mathematics curriculum should reflect this.

* More attention should be paid to making students aware of how mathematics affects their lives, in particular mathematics's role in the development of new technologies.

Thanks for your attention!

Comments and questions are welcomed.