

The Geometry of Urban Services: Theorems, Algorithms, and Proofs

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What is your
definition of
Geometry?

Geometry is a branch of:

Physics

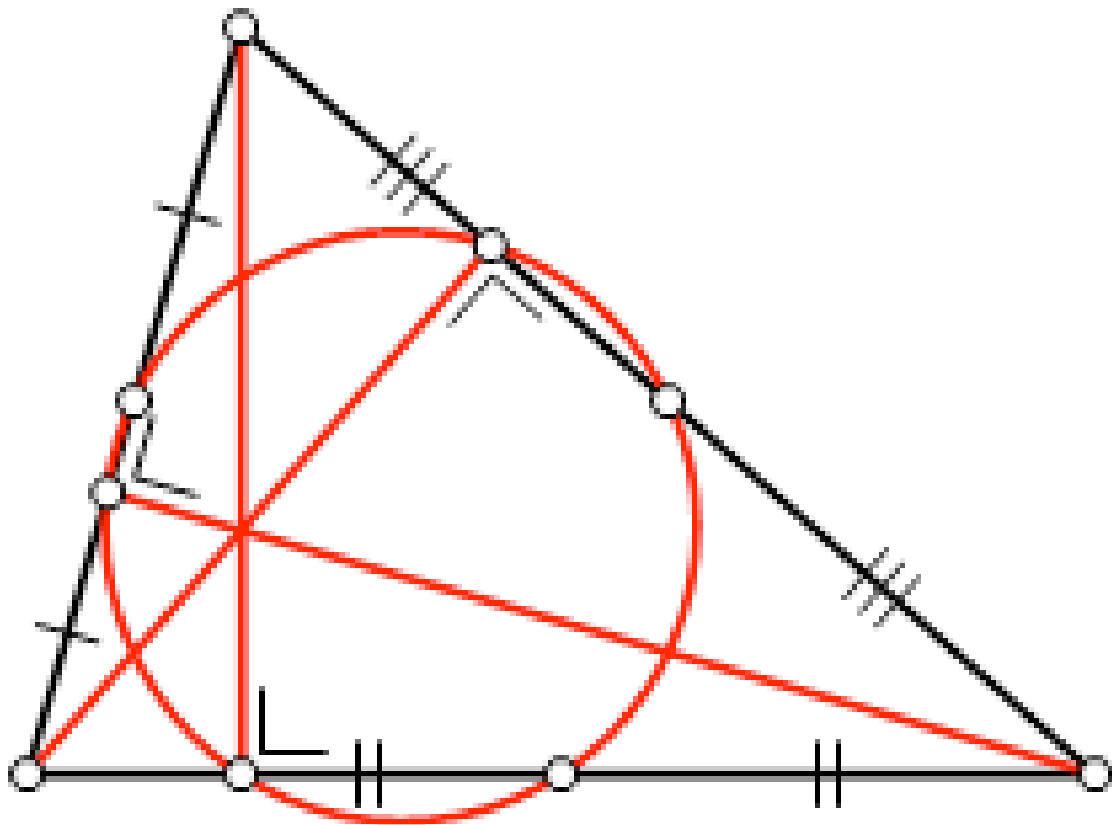
Mathematics

Example:

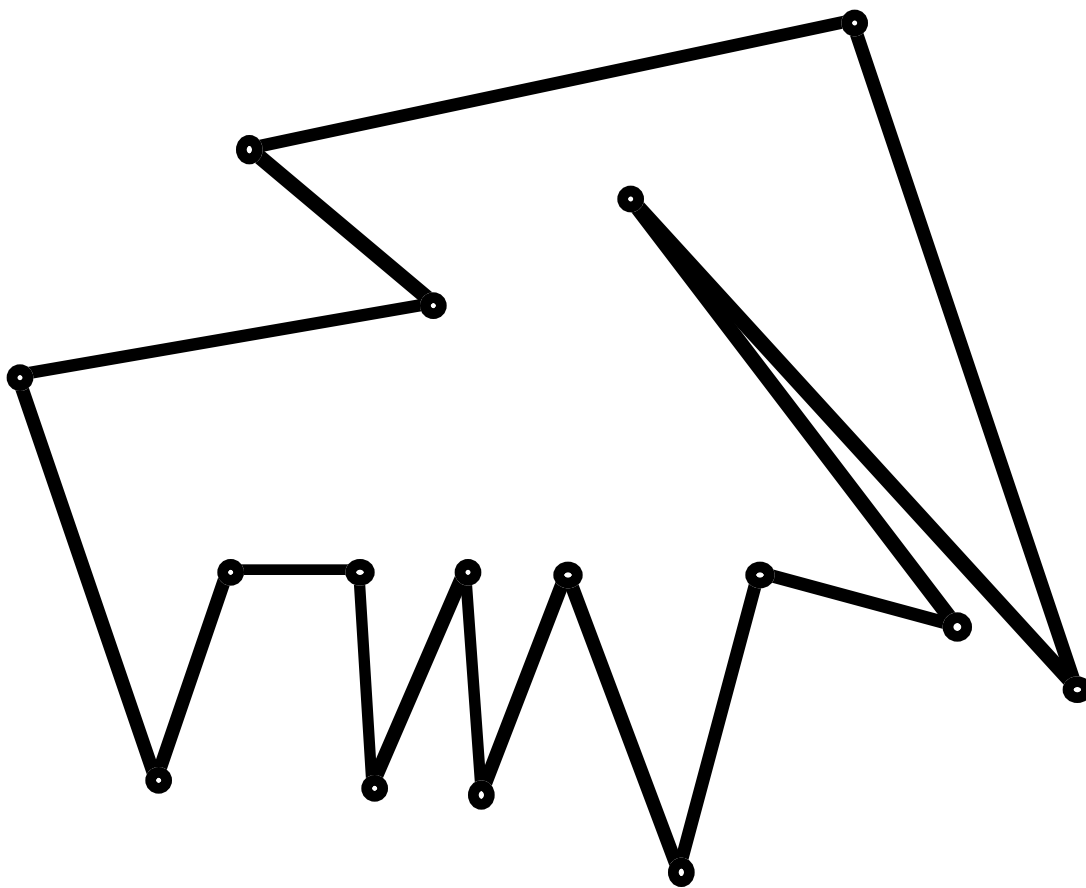
The sum of the
angles of a triangle
is 180 degrees.

Spectacular
geometrical results:

Nine point circle:



How many sensors
located at vertices
will guard this
bank?



How many guards
are sometimes
required and
always sufficient to
guard a polygon
with n sides?

Proofs:

a.

Axiomatic development of
geometry (Euclid)

b.

Proofs based on
"microworld" definitions

Theorems:

(some facts are of interest even if one can not show the proof)

Proof:

(samples of proofs are important)

Algorithms:

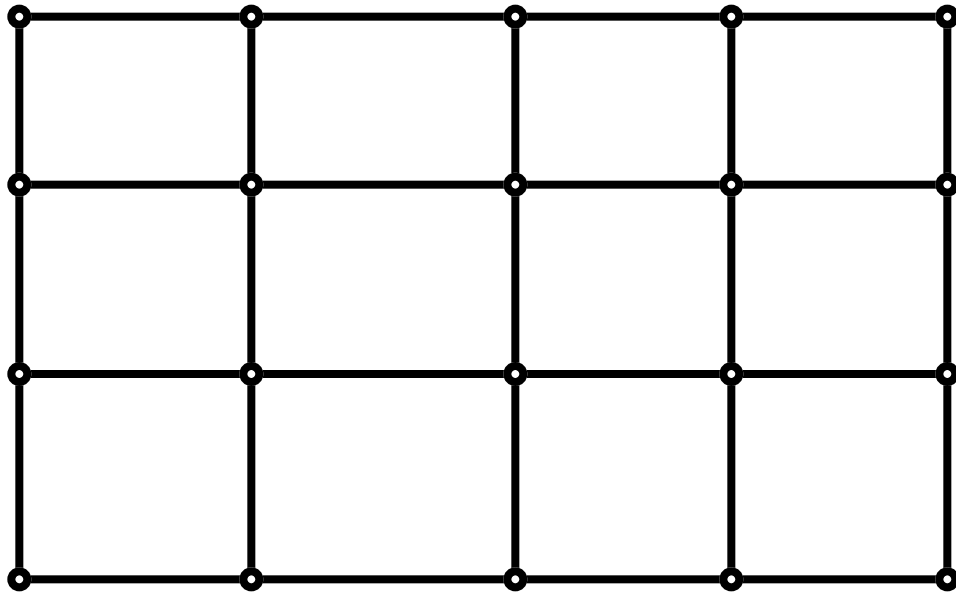
(being able to find what one knows exists is also important).

Geometry =

*The study of
visual
phenomena*

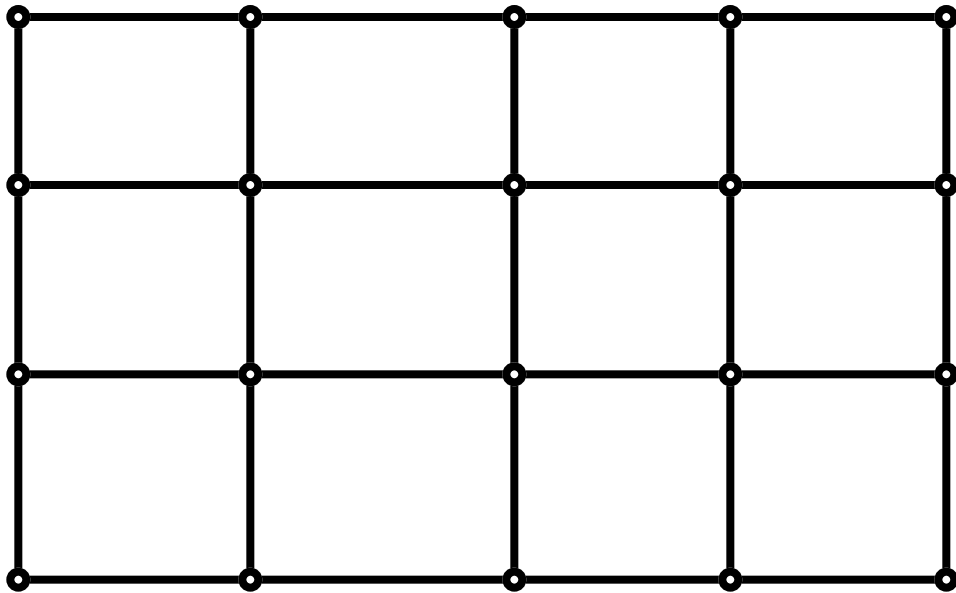
Find an efficient
snow plow route
starting and
returning to
location A:

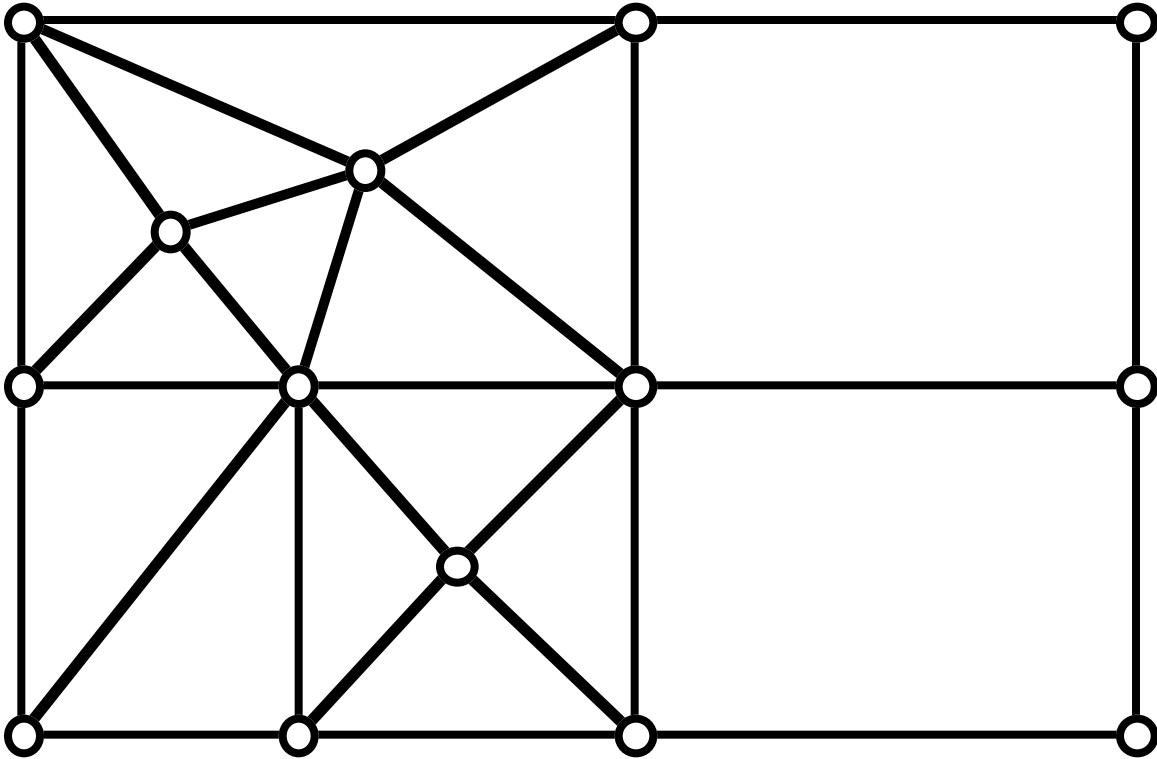
A



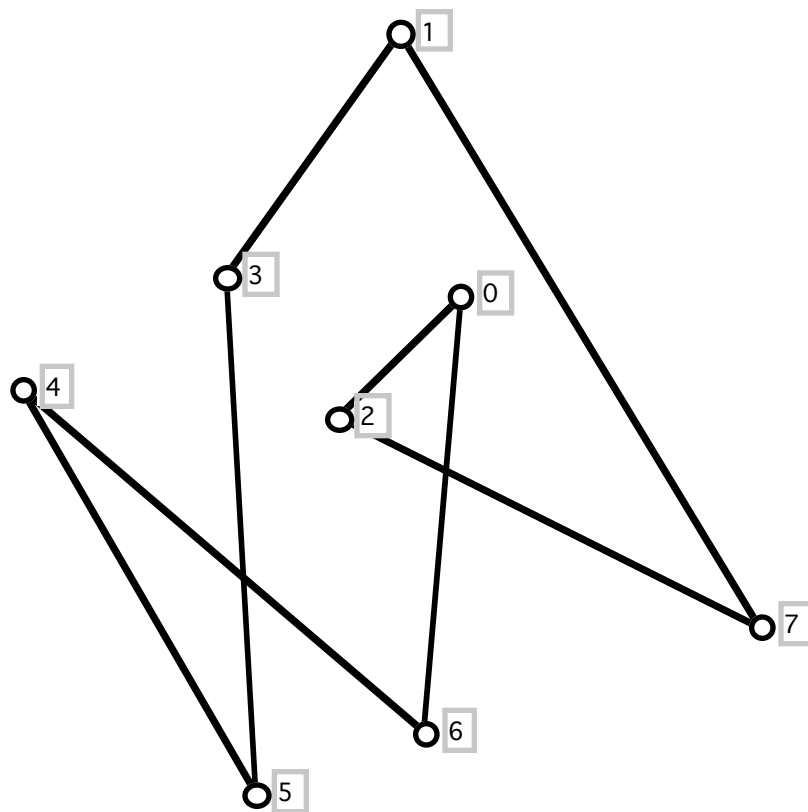
Find an efficient storm sewers (located at corners) inspection route starting and returning to location A:

A





Can the diagram below show the shortest tour of these eight points in the Euclidean plane?

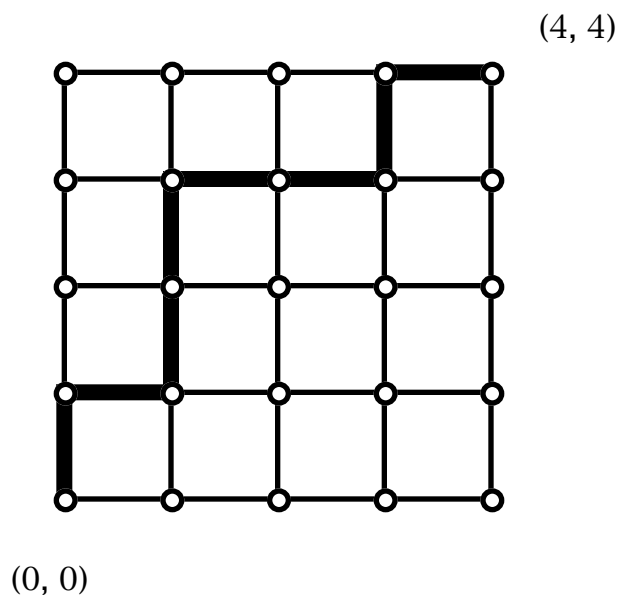


Area:

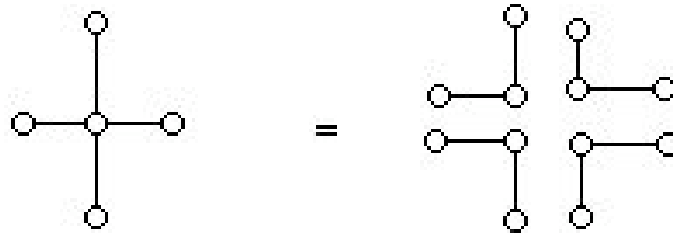
If two polygons P and Q have the same area, can one cut P into a finite number of polygonal pieces and reassemble the pieces to form Q ?

Is the same result true for convex polyhedra?

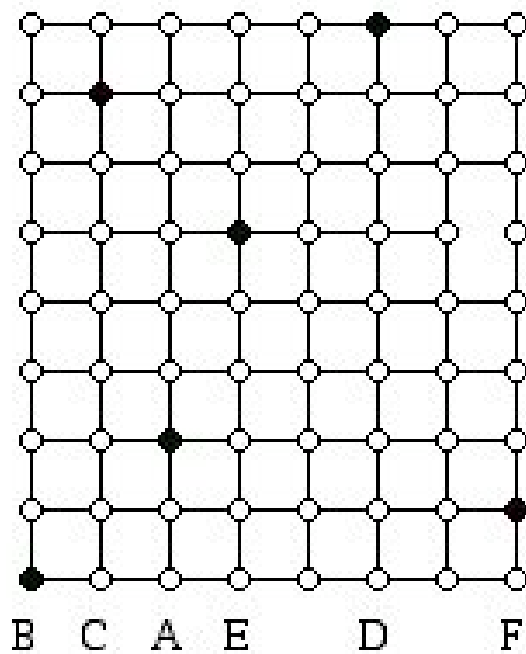
Taxicab Distance



The distance between $(0,0)$ and $(4,4)$ is 8 and there are many paths that achieve this distance.



In the graph below, dots with 4 edges are designed to represent street corners as suggested by the schematic diagram above.



The names associated with the black dots in the grid are shown at the bottom of the grid.

Reference:

*For All Practical
Purposes*

(Developed by COMAP, The Consortium for Mathematics and Applications, for the Annenberg Project of the Corporation for Public Broadcasting, as a book and a Telecourse.)

W.H. Freeman, NY

(Seven editions: Cheap via the internet)