Derivation of the Webster divisor criterion for using a table:

Entries in the table are the original claims divided by:

1, 3, 5, 7, ...

Let us compare the status of states i and j as the seats are distributed to all the claimants.

Using our established notation for population and seats:

If and only if $a_i/p_i > a_j/p_j$ we would say that state i is doing *relatively better* than state j with regard to the current number of seats distributed because state i has MORE representative per person than state j has.

Furthermore we can use the size of $|a_i/p_i - a_j/p_j|$ to measure the "advantage" of one state with respect to the other. If this quantity were zero, the two states have been treated equally; the smaller (closer to zero) this quantity, the less the advantage of one state over the other.

Now suppose, since i has an advantage of j, we remove a seat from i and give it to j. If the inequality as measure above goes down we should do this!

Thus we should look at:

$$|(a_i - 1)/p_i - (a_j + 1)/p_j|$$

$$< |a_i/p_i - a_j/p_j|$$

If this holds we should transfer one seat from state i to state j!

Could it happen that every apportionment might allow such a transfer and that EVERY apportionment was UNSTABLE!

Amazingly, this does NOT happen.

An apportionment will admit NO transfer for any pair i and where

$$a_i/p_i \geq a_j/p_j$$

provided:

$$a_i/p_i - a_j/p_j \leq (a_j + 1)/p_j - (a_i - 1)/p_i$$

Simplifying:

$$a_i/p_i - a_j/p_j \leq (a_j + 1)/p_j - (a_i - 1)/p_i$$

we obtain:

$$p_i/(a_i - 1/2) \geq p_j/(a_j + 1/2) \quad !!$$

-remember that usual rounding is the rounding rule for Webster; we are using the arithmetic mean.)

This means that the table (or rank-index) values for Webster are obtained by dividing the original claims by:

$$1/2, 3/2, 5/2, 7/2 \ldots.$$

and since not the actual values but only their sizes matter, we can divide by:

$$1, 3, 5, 7, \ldots.$$

Similar derivations are used for the other 4 methods (Jefferson, Dean, Huntington-Hill, and Adams) to get the numbers used to prepare the table that implements the rank-index version of the algorithm.

This rank-index approach brings greater clarity to what to do about ties. For the US Apportionment after a Census the Commerce Department publishes the table version of Huntington-Hill for all to see - it has 50 columns, one for each state and enough rows to apportion 435 seats!