

## Stable Marriage Instance, 7 Men, 7 Women (2019)

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Given an instance of the problem of matching  $n$  men and  $n$  women in a stable pairing the Gale/Shapley deferred acceptance algorithm produces either two or one stable marriage arrangement. As the number  $n$  of men and women being matching typically the number of stable marriages grows exponentially but finding all of these stable marriages can't be done using the Gale/Shapley algorithm. However, it is interesting to see the structure of these stable marriages for a particular instance of a fixed size. Below, are provided two tables of preferences, 7 men ranking 7 women, and these 7 women ranking the seven men. Then, the 11 stable marriages that exist for this instance are displayed. Below  $m_3$  will refer to man 3 and  $w_5$  to woman 5. The columns of the table from left to right show a decrease in "satisfaction" from being paired with the woman lists.

Men rank women:

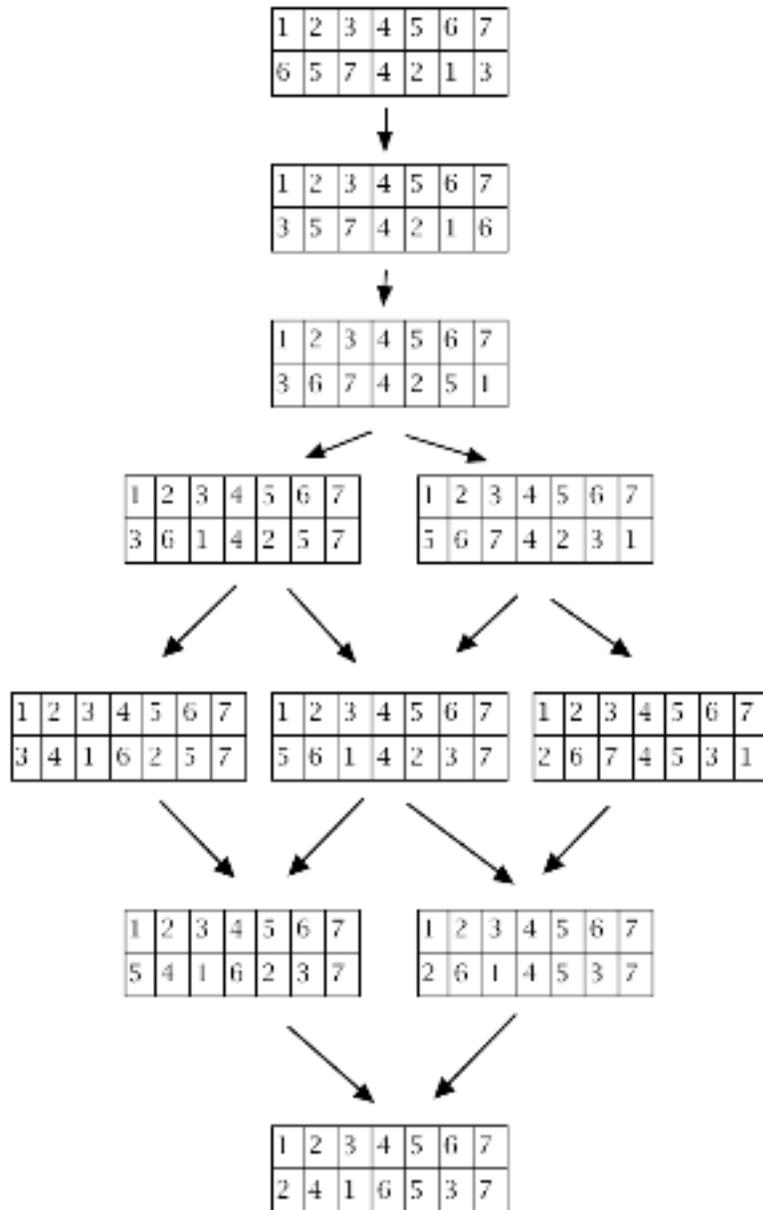
	1st	2nd	3rd	4th	5th	6th	7th
m1	w1	w7	w6	w3	w5	w2	w4
m2	w7	w2	w5	w6	w1	w3	w4
m3	w7	w1	w4	w2	w6	w5	w3
m4	w4	w3	w1	w2	w5	w7	w6
m5	w2	w3	w1	w4	w5	w6	w7
m6	w7	w2	w1	w4	w6	w5	w2
m7	w3	w6	w1	w7	w5	w4	w1

Men rank women:

	1st	2nd	3rd	4th	5th	6th	7th
w1	m3	m2	m7	m5	m6	m4	m1
w2	m1	m5	m4	m6	m3	m7	m2
w3	m3	m6	m1	m5	m4	m2	m7
w4	m7	m2	m3	m4	m5	m1	m6
w5	m5	m7	m1	m6	m4	m2	m3
w6	m4	m2	m3	m7	m6	m5	m1
w7	m5	m7	m3	m2	m4	m1	m6

In the complete list of stable matchings below, the Male Optimal Stable Matching is at the top and the Female Stable Matching is at the bottom. The stable matchings in between progress from the top (most favorable to men) to bottom (most favorable to women) by "trading off" the rank of the mates for men and women compared with the ones in the male and female optimal matching. This raises the issue of whether a good "compromise" might be some "median" position stable matching. One problem with this approach is that there are computational complexity issues in finding a "suitably defined" median stable matching. In many of the applications of the Gale/Shapley approach the "student" optimal rather than the "institution" optimal is used. The appealing part of the Gale/Shapley model is that it is stable and one can prove that when the men propose that no man improve his outcome by lying about his preferences. Furthermore no "group" of men can improve the lot of all men by not being truthful. It is possible, however, in some cases for a group of men to do better for some subset of the men. When the men propose it is possible in some cases for a woman to lie about her preferences and do better.

One approach to finding stable matchings other than the male and female optimal ones, is to start with the male optimal stable matching, pick one of the men, and have him propose, in the framework of the deferred acceptance algorithm to the next woman on his preference list below the woman he is matched with, to see if one can get a new stable matching. Or one can do the analog of this for each woman in the female optimal stable matching.



## References

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