Thought Provoking Election Examples (2020)

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Note: This essay is a work in progress. Some items listed below will be expanded in updated versions.

There are many appealing voting methods that seem very superior to the widely used plurality voting (e.g. the winner is the candidate with the largest number of first-place votes). However, the theoretical work of Kenneth Arrow, Alan Gibbard, Mark Satterthwaite and others suggests that there is no ideal method of obtaining a group decision result from the votes that individuals produce. Over long periods of time many election methods have been suggested but there is also a long tradition of producing specific election examples that are troubling for different reasons. Below is a collection of examples of this kind. Sometimes I know the source of the example but in many cases I don't know the source. Credit is attached where I know the source and thanks to those creative people who developed these examples which help clarify the fairness of different methods. In considering each of the elections, think about your gut reaction to who the winner should be. Then think about who might be the winner using different commonly used methods, or do the calculations necessary to find the winner. Spoiler alert: in some cases I will give the reason that in my view the elections involved are troubling examples.

I will use two notations for ordinal or ranked ballots. The first (Figure 1) is due to Duncan Black and suggests geometrically the ordinal positions of the candidates. When candidates are on the same level they will be listed alphabetically but considered to be of equal preference by the voter. In theoretical discussions about elections often it is assumed voters produce ballots which have no "truncation." Truncation refers to when a voter chooses to rank fewer than the full list of candidates who are competing. Truncation can be done when indifference between candidates is not allowed or it can be done when indifference among candidates is allowed. The reason a voter might truncate his/her ballot may be that some of the candidates are unknown to the voter, or the voter may do it because he/she is voting "strategically." Strategic voting refers to lying about one's true preferences when one completes one's ballot to increase the prospects for candidates you favor. Ranked (ordinal) ballots are used to *rank* the choices/candidates. The other important kind of ballot is a cardinal ballot, which allows the voter to "grade" the candidate/choice on some scale. These different grades can be used to choose a winner or a ranking for the group of individuals who cast the ballots. Important issues for cardinal ballots are what scale to use and whether again, voters can truncate their ballot by not grading all of the candidates. Note that if a scale of 0-99 is used, 0 low and 99 high, a 0 grade for a candidate may reflect that being the grade given to a candidate/choice, the voter knows something about but it may also reflect that the voter knows nothing about this choice/candidate and so can't truly offer a meaningful grade. In this case the voter might choose not to provide a grade for this choice or candidate if this is allowed. However, the choice, if allowed, of not grading some candidate (candidates) might also be for strategic reasons, so as not to harm candidates whom one does grade.

Strategic voting as stated above, refers to when a voter lies about his/her true preference in the hope that this will result in a better outcome from the voter's point of view than voting honestly or sincerely. Sometimes voting strategically can bring about a good result for a specific voter or group of voters but sometimes it can result in an outcome that is worse than if the group voted honestly.

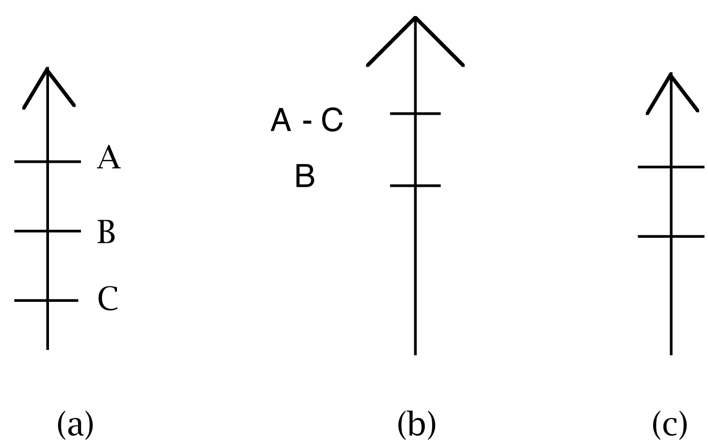


Figure 1 (Ordinal ballots, more preferred candidates towards the top) (a) All candidates A, B, C ranked without indifference: A is preferred to B and C and B is preferred to C. (b) All candidates ranked but A and C are equally preferred, but A and C are both preferred to B. (c) A truncated ballot where though A, B, and C are available to be ranked, the voter has chosen only to indicate that A is preferred to C.)

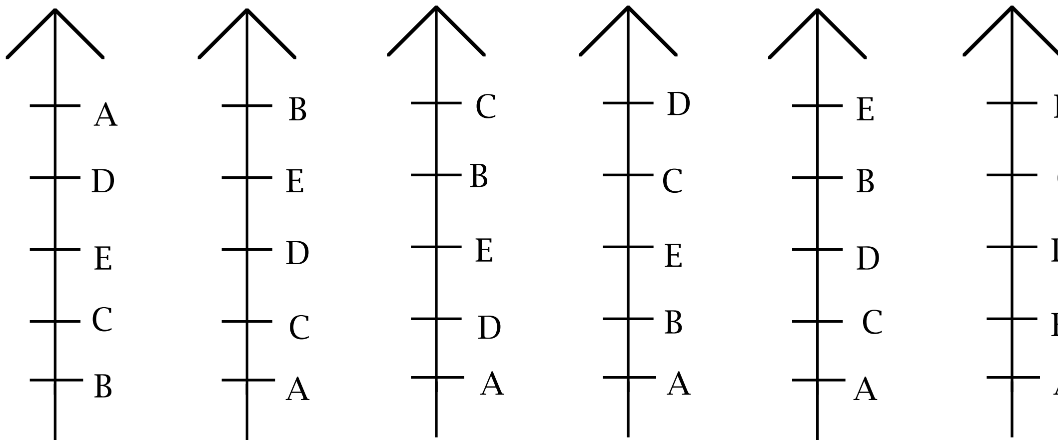
In the second notation the candidates' names appear on the same line and the "greater than" symbol, > , is used to indicate preference. Thus, if X is preferred to Y, X > Y is written. If truncation has been used this is indicated by the fact that not all candidates appear in a line which represents a ballot. When two candidates are considered equally appealing to a voter, this is indicated with an equal sign; usually the candidates ranked equally are listed in alphabetical order. Figure 2 shows the way that the ballots shown in Figure 1 would be coded.

A > B > C	A = C > B	A > C
(a)	(b)	(c)

Figure 2 (Ordinal ballots, more preferred candidates listed further to the left. (a) A is preferred to B is preferred to C. (b) A and C are equally preferred but both of these candidates are preferred to B. (c) A is preferred to C but the voter offers no information about candidate B.)

In the appendix there are some tools for helping with computations that involve using many methods for the same election. Also there is a brief list of methods for deciding elections so that you can compare the outcomes of different methods produced for some of the elections below. We are interested in both the case where a single winner is sought and where a ranking for all of the choices is being sought.

Example 1 (Due to Joseph Malkevitch)

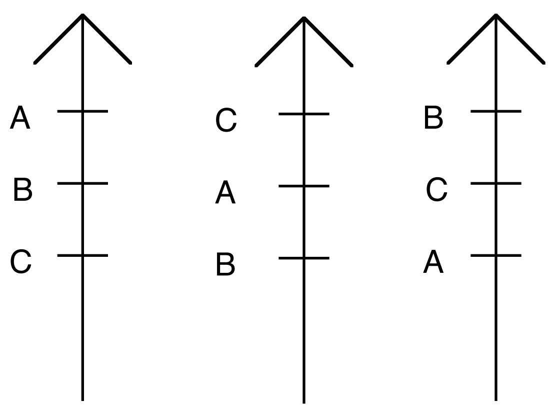


Votes: 18 12 10 9 4 2

Comment:

Five appealing methods of deciding elections produce 5 different winners! What do different methods do in producing a single winner or ranking for this election? Which candidate do you think is most deserving of winning? Many people are surprised that different reasonable election methods can pick different winners.

Example 2



18 21 24 Votes

Comment:

Try using the Condorcet method on this election. Who do other methods select as a winner?

Example 3

49 votes for A > C > B

48 votes for B > C > A

3 votes for C > B > A

Comment:

A is the plurality winner by one vote but a majority of voters rank A last. But C is the first-or second-place choice of all of the voters! C is also the Condorcet winner because C beats each of B and A in 2-way races. What happens in sequential run-off (IRV) which in this example coincides with ordinary run-off? C ge†s only 3 first-place votes and so is eliminated. The three votes for C are transferred to B. In the resulting two candidate election B squeaks out a victory over A! B would not only win by a small margin but 49 percent of the voters considered B the worst choice.

Example 4 (Forest Simmons)

40 A>B>C>D>E
60 E>A>B>C>D

Comment:

This election has ordinal ballots. If one wanted to try to "guess" how this election translates to approval ballots, how would one do that? An approval ballot is a form of cardinal ballot where the scale is 1 and 0 — where 1 means you approve of the choice and 0 means you do not approve of the choice. It is natural to think about what ranking is associated with various cardinal ballots and what grading (cardinal ballots) would be associated with various ordinal ballots.

Note in this election E is a majority winner and a Condorcet winner. But E is the least favorite of 40 percent of the voters in an election with 5 candidates. You may find it of interest to compute the Borda Count winner.

Example 5 (Chris Benham)

Suppose voters are required to produce full ballots with no truncation, and their honest votes are shown below in two versions. First the ballots are cardinal ballots using the scale 0 to 99. Next shown are an ordinal ballot version of the election without "utility" for the candidates given. Third is shown how the voters might vote if truncation were allowed.

Election results:

Cardinal ballots

49 voters with A given 99 points, C given 1 point and B given 0 points
3 voters with C given 99 points, A given 98 points and B given 0 points
48 voters with B given 99 points, C given 2 points and A given 1 point

Ordinal ballots

49 votes A > C > B
3 votes C > A > B
48 votes B > C > A

Truncated ordinal ballots

49 votes A
3 votes C > A
48 votes B

Who do you feel should be the winner in each of these three scenarios?

Who is the winner when you do the computations using the ordinal ballots?

a. Plurality
b. Condorcet
c. Sequential run-off

Who is the winner when you do the computations using the ordinal ballots?

a. Score voting (only applies to cardinal ballot case)

The issue raised here is whether one should solicit intensity of preference information. Can voters do it and do it "accurately?"

Appendix

1. Pairwise preference matrix.

I will describe the pairwise preference matrix for the case where all voters produce full rankings without indifference between candidates. You may want to think about how to modify what is done for other cases, and what the implications are for using the pairwise preference matrix in computing the winner of an election.

Example (Kristofer Munsterhjelm)

This example was developed to show some issues related to the Kemeny method of voting.

Each of the 9 ordinal ranked ballots shown gets one vote:

A>D>F>E>C>B>G
C>A>F>B>E>G>D
C>B>A>F>D>G>E
C>G>E>A>F>B>D
D>F>C>E>B>A>G
E>C>A>D>G>F>B
F>E>G>C>D>B>A
F>G>B>E>A>D>C
G>A>F>C>D>B>E

Below, in the pairwise vote matrix I have suppressed the labels for the rows but the rows correspond to the candidates' names in alphabetical order. The column labels are also omitted but correspond to the for candidates' names in alphabetical order. Note the diagonal entries are indicated with a dash. The table (array, matrix) is symmetrical about its diagonal. The numbers are interpreted as follows: The entry in the ith row and the jth column gives the number of voters who preferred candidate i to candidate j. The (i, j) entry and the (j, i) (when i and j are not the same) add to the total number of people who cast ballots. For example, in this table the number of people who preferred candidate C to candidate G is 6. Since there are 9 voters we can check that the number of voters who preferred G to C is 3. These two numbers can be found in the (3,7) and (7,3) entry of the matrix. Here is a second example: the (5,2) entry is the number of people who preferred candidate E to candidate B, and this entry is 5. The (5,2) entry is 4. There were several pairs of candidates where the number of people who preferred X to Y was only one vote more than the number who preferred Y to X. This is true for candidate A and B and for the pair A and G. On the other hand, B was preferred to C by 7 votes.

Pairwise matrix:

- 5 3 7 4 6 5
4 - 1 4 4 1 4
6 8 - 6 5 4 6
2 5 3 - 4 3 4
5 5 4 5 - 2 5
3 8 5 6 7 - 6
4 5 3 5 4 3 -

Appealing methods for deciding the winner (or ranking) of an election

List of appealing (for very different reasons) election methods. One must specify for all of these methods what to do when a tie occurs, something that is unlikely to happen with examples where there are large numbers of ballots. The methods below are described for the case where each voter ranks all of the candidates, without indifference or truncation. If a method "eliminates" one or more candidates the current ballots are modified to remove this candidate from the voter ballots.

1. Plurality

The candidate with the largest number of first-place votes win.

2. Run-off

If no candidate has a majority, select the two candidates with the largest number of first place votes. The winner of the race between these two candidates is the winner.

3. Sequential run-off (IRV)

If no candidate has a majority, eliminate the candidate with the smallest number of first-place votes. Repeat this process until there is a single candidate left.

4. Borda Count

Given a particular voter's ballet, candidate X is given points for that ballot by counting the number of candidates below X on the ballot. The Borda Count for each candidate is the sum of the number of points that candidate gets from all of the ballots cast. The candidate with the highest Borda Count wins.

Note: There are other systems by which the points can be allocated, some of which yield identical outcomes to what is described above. The system described here works when voters are indifferent in their preference for two or more candidates.

5. Condorcet

If a candidate can beat all of the other candidates in a two-way race, that candidate wins. (For some collections of ballots there is no person who can win two-way races against all of the other candidates.)

6. Anit-plurality

The candidate with fewest last-place votes is the winner.

7. Coombs (Named for Clyde Coombs.)

If a candidate has a majority, that candidate is declared the winner. If no candidate has a majority, the candidate with the largest number of last-place votes is eliminated. Repeat this until one candidate wins.

9. Bucklin

If a candidate has majority counting first-place votes, declare that candidate the winner. If not, for each candidate add the number of first-place and second-place votes. If some candidate has a majority, this candidate is the winner. Note there may be several candidates with a majority at this point. If so, the candidate with the largest majority wins. Repeat until a single candidate wins.

10. Black (Named for Duncan Black, a Scottish economist.)

If some candidate can beat all other candidates in a two-way race, this candidate is the winner. If there is no such candidate, the winner is the candidate who wins using the Borda Count.

11. Baldwin (Named for Joseph Baldwin (1878-1943), an Australian.)

Compute the Borda Count for each candidate. The candidate with the lowest Borda Count is eliminated and the ballots modified to reflect this. Compute the new Borda Count and repeat until one candidate emerges as the winner.

12. Nanson

Compute the Borda Count for each candidate and sum the candidate totals. Divide the result by the number of candidates to get the mean (average) value. Eliminate all candidates at or below the average. If a single candidate remains, that candidate is the winner. If not, delete the candidates' names from the voter ballots who were at or below the mean and repeat the procedure until a single winner emerges.

13. Beat path (Markus Schulze, 1997.)

Beat path elects a Condorcet winner when there is one and some candidate when there is no Condorcet winner. However, it is relatively complicated to explain how the method works to the typical voter. On the upside, it obeys many fairness rules that other methods which elect a Condorcet winner when there is one do not.

Appealing fairness rules for deciding the results of an election

Here is a list of various properties, axioms, or rules that an election method may or may not obey. Much of mathematical election theory has been devoted to sorting out which methods obey which of these "desirable" properties, and to what extent, if one requires a cluster of these properties to hold, there may not be any method which obeys them.

Cloneproof

Condorcet

If there is a candidate who can beat all other candidates in a two-way race, this candidate wins.

Condorcet loser

If there is a candidate who loses to every other candidate in a two-way race, this candidate can't be the winner.

Majority

If there is a candidate who gets a majority of the first-place votes, then this candidate will win.

No favorite betrayal

Ranks equal

Ranks greater than 2

Polynomial time

The ballots can be used to select a winner/ranking using a polynomial time algorithm.

Resolvable

Majority loser

Mutual majority

Smith/ISDA

IIA (Independence of irrelevant alternatives)

Comment: There are many versions of this principle. One version is the following:

If a voting method ranks X above Y when Z is available as a choice, then the method should still rank X above Y when Z is not available as a choice.

LIIA (Local independence of irrelevant alternatives)

Cloneproof

Monotone

Consider two elections. The second differs from the first only in that some candidate has been moved higher on one or more ballots without changing the relative positions of the other candidates. If method M elects candidate C in the first election, then method M should also elect C in the second election.

Consider two elections. The second differs from the first only in that some candidate has been moved down on one or more ballots without changing the relative positions of the other candidates. If method M fails to elect candidate C in the first election, then method M should also fail to elect C in the second election.

Consistency

Reversal symmetry

Reversal symmetry occurs if a method which produces a certain ranking based on the given collection of ballots gives rise to the ranking it initially obtained in reverse order when all of the ballots of the individual voters are reversed.

Later no harm

Later no help

Burying

Participation

No favorite betrayal

Summable: O(N!)

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