

Game Theory Activity 2 (2021)

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1. The diagram below shows an example of a matrix game with players Row (R) and Column (C). The entries in the table "code" the payoffs to the players when the game is played based on the actions of the players.

	Column I	Column II
Row 1	(3, -3)	(-2, 2)
Row 2	(6, -6)	(2, -2)

Figure 1

The game is played as follows. Row gets to choose between Row 1 or Row 2 to "play," while Column gets to play (choose) either Column I or Column II. Row and Column make their choices without consulting each other. One can think of the players as writing down their choice on a piece of paper which they hand to an umpire (referee). The referee consults the two "actions" by the players and supervises the "payoff" between the players. The entries in the matrix (table) give the payoffs to the players. For simplicity we will assume the payoffs are in money - pennies. Thus, if Row plays Row 2 and Column plays Column II then Row wins 2 pennies and Column loses 2 pennies. Row's payoff is the first entry in the pair shown in the matrix and Column's payoff is the second entry. In this game the sum of the entries in each pair adds to zero, so the game is known as a zero-sum game. We will think of Row's gains as coming at Column's expense and Column's gains as coming at Row's expense. Thus, the sum of the payoffs to the two players adds to zero,

which is why games of this kind are called zero-sum games.

Play this game 6 times against someone where you are Row and your opponent is Column. Then play the game again 6 times where you interchange the roles of playing Row and Column. What do you learn by playing this game? Would you rather be Row or Column?

2. The diagram below shows an example of a matrix game with players Row (R) and Column (C), where the rules of play are similar to the prior example.

	Column I	Column II
Row 1	(3, -3)	(-1, +1)
Row 2	(-9 +9)	(+3, -3)

Figure 2

Play this game 6 times against someone where you are Row and your opponent is Column. Then play the game again 6 times where you interchange the roles of playing Row and Column. What do you learn by playing this game? Would you rather be Row or Column?

Comment: These games are known as *matrix games* and are called 2x2 games because Row and Column can each choose two actions.

Some things to think about:

- When are games such as those above fair?
- Can you generalize what you learned from the game in Figure 1?
- Can you generalize what you learned from the game in Figure 2?
- What does the role of the number of times the game is played have in the analysis of games of this kind?
- What advice would you give to Row and Column about how to play the game in Figure 1?
- What advice would you give to Row and Column about how to play the game in Figure 2?

3. Games involving two players (Row and Column) which involve flashing a certain number of fingers by each player and getting a payoff depending on the pattern of fingers displayed are popular on New York City streets, though more often than not, to decide who goes first in some street game rather than as below, where actually "money" changes hands. Suppose Row can display one or two fingers, and Column can display one or two fingers. If the sum is even then Column pays Row the size of that sum in pennies. If the sum is odd Row pays Column that sum.

(a) Thinking of this as a zero-sum game with two players, construct a payoff matrix in the style above for this game. (Hint: What payoffs are possible for each play of the game?)

(b) Do you think this is a fair game (e.g. when played many times neither player wins nor loses money)? If so, why?

(c) Play the game 10 times. Do you think the game is fair after actually playing the game? Why?

(d) Determine an optimal way to play this game for each of Row and Column assuming the game is played many times.