Bankruptcy Models (Spring, 2021)

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When there is a pot of money which exceeds the claims against it, there are fascinatingly many different approaches to settling the claims, which are based on the many ways that one might try to be fair, but are not always compatible with each other.

Below I look at a few examples, with no attempt to be complete, but to serve as a way to check via examples the different results when different approaches to solving bankruptcy problems are used.

First, consider a two-player (claimant) example. There is an estate E of size 100 with A having a valid claim of 10 and B having a valid claim of 190. Note that B claims more than the "estate" (pot to pay off the claims) E. Some theorists about bankruptcy problems have argued in favor of "truncating" claims exceeding the estate to the size of the estate before trying to settle the claims, but I will not use that approach here.

The methods used will be:

1. Entity equity (allows a claimant to get more than the claimed asked for)

2. Maimonides gain (equalize as much as possible the gains given to the players)

3. Loss equity (with subsidization - some players may have to add to E with their own money)

4. Maimonides loss (equalize as much as possible the losses incurred by the

claimants)

5. Proportionality

6. Shapley value

7. Concede and Divide (contested garment rule) (applies only for 2 claimants)

8. Talmudic Method (applies to 2 or more claimants)

Example 1 (Two claimants)

A = 10   B = 190   Estate = 100   (Collective loss: 10 + 190 - 100 = 100.)

1. Entity equity:

A gets 50 and B gets 50.

Note: A gets more than A asked for, and B has a loss of 140.

2. Maimonides gain

(Equalize what is distributed as much as possible but do not give claimant more than the person claimed.)

A gets 10 and B gets 90. (A gets his/her total claim but B loses 100.)

3. Equalize loss (if necessary some claimants may have to subsidize the settlement)

One approach to solving this case with 2 claimants leads to the problem of solving two linear equations in two variables ("unknowns").

Suppose A gets a and B gets b (a notation sometimes used further below).

We have the equations:

$a + b = 100$

A's loss is 10 - a and B's loss is 190 - b, and these quantities are equal:

So:

$a + b = 100$
$10\text{-}a = 190\text{-}b$

$a+b = 100$
$-a + b = 180$

So $2b = 280$
$b = 140$ and .and $a = -40$

Check:

A's loss is 10 + 40 = 50. A's loss is 190 - 140 = 50, so both their losses are equal.

Easier solution: Since total loss is 100, we make each claimant have a loss of 50.

This will mean giving a = -40, requiring a subsidization of 40 for a total loss of 50 and giving b a loss of 50 means that he is given 140, 100 from the estate E and 40 more that A provides.

4. Maimonides loss (tries to equalize the loss as much as possible)

Initially A has a loss of 10 and B has a loss of 190. As we pay B money from the estate, B's loss starts to move in the direction of A's loss of 10. When B is given 100 there is no more to give:

$a = 0$ and $b = 100$

Thus, A lost 10 and B lost 90, but this is the best one can do without A providing money to the settlement.

5. Proportionality

The total of the claims adds to 200 (10 + 190 = 200).

So A gets 10/200 or 5% of 100 which is 5. B gets (190/20)(100) = .95(100) = 95

Check: 5 + 95 = 100

6. Shapley (average over permutations read from left to right; with each player given his claim, until the estate is exhausted)

AB   A gets 10 and B gets 90
BA   B gets 100 and B gets nothing.

A's final payment is (10 + 0)/2 = 5; B's final payments is (90 + 100)/2 = 95.

7. Concede and Divide (each claimant is given his/her uncontested claim and the remaining amount is divided equally between them.)

A's uncontested claim against B is 0. B's contested claim against A is 90

A is given 0 + (10)/2 = 5
B is given 90 + (10)/2 = 95.

8. Talmudic method (pay off half-claims using Maimonides gain and if there is part of the estate left distribute this to pay off the remaining half-claims using Maimonides loss.)

Half-claims are:

A = 5 and B = 95, which can be met by giving a = 5 and b = 95, which is the whole estate.

So the algorithm now terminates with:

a = 5 and b = 95

Example 2   (Three claimants)

A = 30   B = 60 C = 90   E = 120

(Note, collective loss is (30+60+90) - 120 = 180 - 120 = 60

1. Entity equity:

A gets 40, B gets 40 and C gets 40. (A gets more than he/she asked for.)

2. Maimonides gain

A = 30   B = 45 and C = 45

Initially, all claimants can be given 30 because 30(3) is less than 120, the amount E available. Now, since 120 - 90 = 30 is still left, we can give B and C equal amounts of 15, without exceeding B or C's claim.

3. Loss equity

Since there is a loss of 60 to be shared equally, we give:

a = 10 b = 40 c = 70 Which means each of the claimants loses 20.

4. Maimonides loss

In this example, equity loss and Maimonides loss give the same result:

a = 10 b = 40 c= 70

5. Proportionality

Since the total claims add to 180, the proportional shares are:

A's claim of 30/180 is 1/6 and since E is 120, A gets 20.

B's claim of 60/180 is 1/3 and since E is 120, B gets 40.

C's claim of 90/180 is 1/2 and since E is 120, C gets 60.

As a check we see that 20 + 40 + 60 is 120, the size of the estate E.

6. Shapley (Initial problem: A = 30 B = 60 C = 90; E = 120

There are 6 permutations to read from left to right:

ABC so A gets 30, B gets 60 and C gets 30 (30 left after A and B get their 90)

ACB so A gets 30, C gets 90 and B gets 0

BAC so B gets 60, A gets 30 and C gets 30

BCA so B gets 60, C gets 60 and A gets 0

CAB so C gets 90, A gets 30 and B gets 0

CBA so C gets 90, B get 30 and A get 0

A's final allotment is (1/6)(30+30+30+0+30+0) = 120/6 = 20

B's final allotment is (1/6)(60+0+60+60+0+30) = 210/6 =35
C's final allotment is (1/6)(30+90+30+60+90+90) = 390/6 = 65

Check: 20+35+65=120 as required.

7. Concede and Divide

This method does not apply to 3 claimants.

8. Talmudic Method

First we list 1/2 the claims:

A = 15 B = 30 C = 45

We try to pay these off using Maimonides gain. We see that these can be met using E = 120, with 30 left over because 15 +30 + 45 = 90

Now we have the 1/2 claims of:

A = 15 B = 30 C = 45

Applying Maimonides loss we give C 15 to reduce C's claim to 30. Now we have 15 units left so we can give B and C a 7.5 reduction in loss.

In this phase: A gets 0, B gets 7.5 and C gets 22.5.

So the total allocation from both phases is:

A gets 15 + 0 = 15; B gets 30+7.5=37.5; C gets 45+22.5= 67.5

Check: 15 + 37.5 + 67.5 = 120 as required.

Let us check one of the "consistency" calculations for the Talmudic Method for a subset of the players, say, B and C.

B and C together get 105.

If we look at the claims B = 60 and C = 90 with an estate of 105, what are the payoffs using concede and divide?

Applying concede and divide:

A's uncontested claim against B is 15; B's uncontested claim against A is 45. These add to 60 so there are 45 units they share equally.

B gets 15 + 22.5 while C gets 45 + 22.5. So B should get 37.5 and B should get 67.5. And this, indeed, is what they got using the Talmudic Method.

Remember that the Talmudic Method and Concede and Divide give the same results for two players.

Bibliography

The leading expert on bankruptcy models is William Thomson at the University of Rochester (Economics). Many of his papers are freely available on the web and he has written an important book about this topic:

Thomson, William. How to divide when there isn't enough. No. 62. Cambridge University Press, 2019.