Dominating Strategy Analysis in a Zero-Sum Game: Activity (2021)

Prepared by:

Joseph Malkevitch
Mathematics Department
York College (CUNY)
Jamaica, NY 11451

email:

malkevitch@york.cuny.edu

web page:

http://york.cuny.edu/~malk

Consider the zero-sum game that is shown in Figure 1 involving two players Row and Column. Row can take any one of 4 actions and Column can take any of 4 actions. The payoffs are shown in Figure 1. There are 16 possible outcomes and each outcome is represented by a payoff pair where the two elements of the pair add to zero, confirming that we have the earnings of Row coming at the expense of Column (or vice versa). The pair arising from Row action 3 (play Row 3) and Column action IV (play Column IV) is (3,-3) and we will always, when using this notation, list the payoff for Row first, the first coordinate of the pair, and the payoff for Column second, the second coordinate of the pair.

	Col I	Col II	Col III	Col IV
Row 1	(1,-1)	(1,-1)	(-3,3)	(-4,4)
Row 2	(3,-3)	(2,-2)	(-2,2)	(-3,3)
Row 3	(-5,5)	(4,-4)	(4,-4)	(3,-3)
Row 4	(2,-2)	(-1,1)	(-5,5)	(-4,4)

Figure 1 (A zero-sum game; 4 actions (moves) are available for each player each time the game is played.)

Problem 1:

(a) Simplify this game as much as possible by eliminating dominating rows and/or dominating columns.

Note: Remember that initially there may be no dominating row(s) (or column(s)) but new dominations may appear after some rows or columns are eliminated.

(b) If possible, find the "value" for the game and the optimal strategies for both players.

It is traditional in writing down zero-sum games to show the payoffs from Row's point of view since one can deduce Column's payoffs from what row gets - if Row's payoff is -11, then Column wins 11.

Figure 2 shows the same game as in Figure 1 but showing the payoffs only from Row's point of view.

	Col I	Col II	Col III	Col IV
Row 1	1	1	-3	-4
Row 2	3	2	-2	-3
Row 3	-5	4	4	3
Row 4	2	-1	-5	-4

Figure 2 (A zero-sum game; 4 actions (moves) are available for each player each time the game is played. Payoffs from Row's point of view.)

Problem 2:

(a) Simplify this game as much as possible by eliminating dominating rows and/or dominating columns.

Note: Remember that initially there may be no dominating row(s) (or column(s)) but new dominations may appear after some rows or columns are eliminated.

(b) If possible, find the "value" for the game and the optimal strategies for both players.

Comment: You should get the same answers for Problem 2 as for Problem 1.

Comment: Working with Figure 2 often helps students understand the nature of absolute value and signed numbers better. If in a column there are a 4 and a -2 in different rows, then Row prefers the 4 to the minus 2. But if there are a 4 and a -2 in different columns, then Column prefers the -2 to the 4 because the -2 means Column wins and the 4 means that Column loses!