

Is IRV (Sequential Run-Off) Better than Plurality? (2021)

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New York City recently changed its voting procedure for local elections starting in Nov. 2021. IRV (Instant Run-Off Voting), which I prefer to call with the more suggestive name sequential run-off voting, will replace plurality voting. In particular, the standard ballot (vote for one) will be replaced by ordinal (ranked) ballots. The discussion below assumes a prior minimal knowledge with voting methods.

The following election due to Brian W. Goldman is thought provoking. The election has 5 candidates whose names are C for centrist, L for leftist, R for right wing, FL for far left and FR for far right. For simplicity of calculations we keep the number of voters small.

The 24 voters produced the ballots shown below:

7 votes: $C > L > R > FL > FR$

6 votes $L > C > R > FL > FR$

6 votes $R > C > L > FR > FR$

2 votes $FL > L > C > R > FR$

3 votes $FR > R > C > L > FL$

Note that with no truncation of ballots or indifference the voters produced only 5 of the 120 possible ballots that one could produce of this kind. You

can decide if you think if these are plausible ways of voting for candidates based on a scale from far right to far left. Thus, if a voter produced the ballot below:

FR > FL > R > L > C

some might think this ballot peculiar (or perhaps this is the way you view the world) but our framework is not to second guess what voters do. Any proper ballot will be counted.

Before reading what appears below decide who you think "DESERVES" to win based on the ballots that the voters produced. Generally there is a view that voting, i.e. each voter in secret casting a ballot on the issues, can be "translated" from the voices of individuals to a "voice of the people." But mathematics shows that this is naive.

So with plurality voting since the Centrist C got the largest number of votes, C wins.

To study the election further consider the pairwise preference matrix for this election, which is displayed in Figure 1 below. The entry in Row L and Column FL means that 22 voters preferred candidate L to FL and the entry in Row FR and Column C means 3 voters preferred FR to C. Note also that there is one extra column which gives the row sums.

	C	L	R	FL	FR	Row Sum
C	-	16	15	22	21	74
L	8	-	15	22	21	66
R	9	9	-	22	21	61
FL	2	2	2	-	15	21
FR	3	3	3	9	-	18

Figure 1 (Table showing voter preferences between candidates. The entry of 2 in row FL and Column L means 2 voters preferred the Far Left candidate to the Left candidate while the 22 in row C and column FL means that 22 voters

preferred the Centrist candidate to the Far Left candidate.)

The Table in Figure 1 is useful in showing who is the winner for these ballots with a variety of methods.

Who is the winner using various appealing methods for deciding the winner of an election using ordinal ballots?

1. Plurality (largest number of first-place votes)

Plurality winner is C (Centrist).

Note: This is a close election because C has only one more vote than L and R.

2. Condorcet (Candidate if one exists who can win two-way races against all of the other candidates.)

Here is where the Table in Figure 1 comes in handy. We can read off from that table, looking at the first row of numbers and the first column, that C beats all the other candidates in a two-way race.

Condorcet winner is C.

3. Borda Count (Candidate with largest number of points wins where the number of points for candidate i from a particular ballot is the number of candidates ranked below i .)

Comment: While it may not be initially obvious, the Table in Figure 1 can be used to compute the Borda Count. From first principles of the definition, how many points does C get? The answer is given in the line below:

$$C = 7(4) + 6(3) + 6(3) + 2(2) + 3(2) = 74.$$

Is it an accident that the sum of the entries of the first row in the Table of Figure 1 is 74? No!! It turns out to be a theorem for the case of ranked ballots with no indifference or truncation. Using this theorem, C is the Borda winner.

Borda Count winner is C.

4. Run-off (If no one gets a majority, the winner is the person who wins the two-way race between the candidates with the largest number of first-place votes, eliminating the more poorly performing first-place vote candidates.)

A complication in this example arises because there is a tie for who has the largest number of second-place votes. The outcome of the election might depend on how this tie is broken, but in this example, whichever way the tie is broken still leads to the same candidate winning the run-off.

Run-off winner is C.

5. Bucklin (If a candidate has a majority counting first-place votes, declare that candidate the winner. If not, for each candidate add the number of first-place and second-place votes. If some candidate has a majority this candidate is the winner. Note there may be several candidates with a majority at this point. If so, the candidate with the largest majority wins. Repeat until a single candidate wins.)

Bucklin winner is C.

Ok, all these appealing methods seem to make C the winner.

But this example, when decided by the new voting method that NYC will use in local elections starting in 2021, does not result in the winner being C!!

6. Sequential run-off (IRV = Instant run-off voting) (If no candidate has a majority, eliminate the candidate with the fewest first-place votes, and conduct an election with the remaining candidates. Repeat the procedure until one candidate emerges as a winner.

Round 1:

C gets 7 votes, L gets 6 votes, R gets 6 votes, FL gets 2 votes, FR gets 3 votes.

FL has the fewest first-place votes, 2, so FL is eliminated. The 2 voters who voted for FL over all the others list L as their next choice, so in the next round L will get two more votes.

Round 2:

C gets 7 votes, L gets 8 votes, R gets 6 votes and FR gets 3 votes.

Since FR gets the fewest votes, FarRight is eliminated and those three votes now get transferred to R.

Round 3

C gets 7 votes, Left gets 8 votes and Right gets 9 votes.

Since C has the fewest first-place votes, C gets eliminated and C's 7 votes can be transferred to L.

Round 4

L gets 15 votes and R gets 9 votes so L wins!

Note, sequential run-off yields a different winner from all the other methods so far. Does L being winner accord with your "gut" view of who should win or who represents the "will of the people?"

There are "modifications" of IRV (sequential run-off) that eliminate candidates with few first-place votes but do checks to make sure that if there is a Condorcet winner, that candidate is not eliminated.

Burlington, Vermont like many America cities historically used Plurality voting with a run-off if no candidate got at least 40 percent of the vote but for the period 2006-2010 used IRV (sequential run-off). In 2010 the use of IRV was repealed after an election was held in which, like the election above, a candidate who got the most first-place votes and was, in fact, a Condorcet winner, LOST the election. Burlington is now considering returning to IRV (sequential run-off).