Example to Contrast Adams, Jefferson {D'Hondt), Webster (Saint-Laguë) (2021)

Prepared by:

Joseph Malkevitch
Department of Mathematics
York College (CUNY)
Jamaica, New York 11451

email:

malkevitch@york.cuny.edu

web page:

http://york.cuny.edu/~malk/

The table (rank index) method is shown below for a "toy" example that illustrates some of the issues with regard to the apportionment problem.

In this example we have three states (parties) and they have claims against 10 seats. The claimants have claims:

A = 630   B = 320   C = 50

In the tables below, decimals are rounded to the nearest 1/10. One can think of the numbers above as populations of "states," or votes for "political parties." We need to give A, B, C positive integer numbers of seats adding to h (here, 10). Sometimes there is the additional condition(s) that each state get at least one seat or that parties with too small a vote don't get representation.

Note that for this example: A has 63%, B has 32% and C = 5% of the raw number totals. Since the total claims add to 1000, and h = 10 the "exact quota" is 100. Thus, 63% of 10 is 6.3 and 6.3 is also the quotient of 630/100. For European apportionment problems remember that small parties are not guaranteed seats in the parliament but typically if a party got as much as 5% of the vote it would not be ruled out for getting a seat. Saint-Laguë does not guarantee a party seats but depending on the relative sizes of the party votes some parties might not be able to get a seat. However, for the Adams method all claimants are guaranteed at least one seat by the methodology of the method. When h is smaller than the number of parties for Adams some method must be developed to break the inevitable tie.

D'Hondt

h=10	A	B	C
Original claims	630	320	50
Divide by: 1	630 1	320 2	50
2	315 3	160 5	25
3	210 4	106.7 8	16.7
4	157.5 6	80	12.5
5	126 7	64	10
6	105 9	53.3	8.3
7	90 10		
8			

Comment: This table can be used to assign 1 to 10 seats based on h = 10 and the given claims for A, B and C for D'Hondt. However, to decide who would get an 11th seat (h=11) would require filling in the row of the table where division by 8 is used. Since 630/11 is 78.8, we would give the 11t h seat to B rather than to A.

Note also that using Jefferson's method we would have to give C a seat at the start, whereas with D'Hondt, even with h = 11, C would not get one seat.

What is the exact quota here? 1000/10 = 100, So the "fair share" for A, B, and C are 6.3, 3.2, and .5. So the Hamilton Apportionment would be A = 6, B = 2 and C = 1. (See below for the Webster method analysis.) However, D'Hondt gives A = 7 seats and B = 3 seats out of the h = 10. Note that using D'Hondt, C gets no seats in this example when the house size is 10. By extending the table you determine the smallest house size h where C does get a seat!

Saint-Laguë (Webster's method): (Note: C eventually gets one seat, but not automatically at the start as would be true for Webster.)

h=10	A	B	C
Original claims	630	320	50
Divide by: 1	630 1	320 2	50 10
3	210 3	106.7 5	16.7
5	126 4	64 8	10
7	90 6	45.7	
9	70 7	35.6	
11	57.3 9	29.1	
13	48.5		

As I use the term Webster's method automatically assigns each state one seat. Here I show the way Saint-Laguë, otherwise equivalent to Webster which gives one seat to each claimant) works for this example. For Webster the first three seats would be "shared by A, B, and C. The 10th seat assigned would go to A. The results would be that A gets 6 seats, B gets 3 seats and C gets one seat. Note that if we had used the divisor approach. A's fair share is 6.3, B's 3.2 and C's .5. Usual rounding yields, A = 6, B = 3, and C = 1.

Note the next page shows the order of the seats being assigned using Webster because each state initially must get one seat.

This is the table for Webster where initially each claimant gets one seat. If the house size is large enough (and the number of claimants relatively small compared that house size) the Saint-Laguë and Webster table eventually assign the additional seats in the same way.

h=10	A	B	C
Original claims	630	320	50
Divide by: 1	630 1-3	320 1-3	50 1-3
3	210 4	106.7 6	16.7
5	126 5	64 9	10
7	90 7	45.7	
9	70 8	35.6	
11	57.3 10	29.1	
13	48.5		

Adams Method

Note we use the symbol for infinity ∞ to code the fact that Adams automatically gives claimants each one seat. So some system must break ties when there are more claimants than seats for this method.

h=10	A	B	C
Original claims	630	320	50
Divide by: 0	∞ 1-3	∞ 1-3	∞ 1-3
1	630 4	320 5	50
2	315 6	160 8	25
3	210 7	106.7	16.7
4	157.5 9	80	10
5	126 10	64	8.3
6	105		
7			

Note that A, B, C share the first 3 seats, as it were, because each claimant automatically gets a seat with Adams. In the end A's share is 6, B's share is 3 and C gets 1 seat. Note the subtle difference in the order in which the seats are assigned because C got a seat at the start. We can see who would get seat 11 if h =11 but for more than h = 11 we would need more rows in the table. Note that with D'Hondt C got no seat when h = 10 holds.

It was initially the work of E. V. Huntington that showed that divisor and table methods (rank index) were "equivalent." There are continuing disputes about the extent to which Hamilton, Huntington-Hill, Adams, Jefferson {D'Hondt), and Webster (Saint-Laguë) treat small (large) states (parties) fairly, that is show bias based on the size of their claims.

Reference:

Balinski, M. and H. Young, Fair Legislative Representation, (2nd edition), Brookings Institution, Washington, 2001.