

# **Syllabus for Mathematical Game Theory 5017 and Advanced Topics in Mathematical Game Theory 6017: Spring 2022**

**Instructor:**

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**Special Attention will be given to fact that after the 2020 Census the House of Representatives will be apportioned as specified by Congressional Law and the Constitution. This not only involves the use of the currently used Huntington-Hill Method but also the possible gerrymandering of districts for the states by state legislatures. We will also treat the consequences for NYC Elections that ranked ballots are being used in both primaries and elections as well as that the ballots are counted using a "version" of IRV, instant run-off voting.**

## **SYLLABUS (Mathematical Game Theory, Spring 2022)**

0. What is Game Theory? (Mathematics has provided insight into two types of games - situations which involve "conflict" or the making of decisions: i. combinatorial games include chess, checkers, Nim, Hex, etc. and ii. political and economic games including Prisoner's Dilemma and Chicken. We will deal primarily with the latter but will also deal with related "fairness" topics such as elections and voting, two-sided markets, bankruptcy, fair division, and apportionment. In particular we will discuss game theory aspects of the primaries being held in 2019-2020 and the 2020 US elections (all 435 members of the House of Representatives were selected as well as about 1/3 of the Senate) and a new President will be elected. In 2020 there will be a census of the US population which raises issues about new legislative district lines for 435 House of Representatives districts, and the gerrymandering of these districts. Often the issue is to understand what it means to be fair and/or behave in a rational fashion in a wide variety of contexts.

1. Taxonomy of games (Number of players, zero-sum games, non-zero-sum games, cooperative and non-cooperative games. Utility: transferable and non-transferable utility. Role of information/chance. Values for games: equilibrium concepts, Shapley value.)

2. 2-person zero-sum games (Matrix games, pure strategies, using spinners to find optimal mixed strategies; fair games; connections with linear programming. Applications in economics and political science.)

3. 2-person non-zero sum games (Prisoner's Dilemma, Chicken, Nash equilibria, congestion games; Braess's Paradox, price of anarchy. Rationality - connections to behavioral and experimental economics.)
4. Two-sided markets (Gale/Shapley models) (Matchings. Stable marriage. Gale/Shapley models, deferred acceptance algorithm; male optimal - female optimal stable solutions; school choice and other applications, kidney exchange.)
5. Elections and voting (Plurality, run-off, Instant run-off elections, Borda count, Condorcet methods, approval voting. Arrow's Theorem.) Contrasting ranking systems with grading systems (Majority judgment).
6. Bankruptcy (Proportionality, Maimonides gain and loss, the concede-and-divide rule and the talmudic method.)
7. Apportionment (Hamilton's, Jefferson's, Webster's and Huntington-Hill's methods. House and population monotonicity. Balinski-Young Theorem.)
8. Combinatorial Games (Nim, graph games. Nim addition and hackenbush.)

Note: Game Theory is not a standard topic in the K-12 curriculum. This course will show how game theory topics can be infused into the usual curriculum (including Standards-based curriculum) as ways of illustrating results in arithmetic, algebra, geometry, probability and statistics.

Text: (No one book covers what will be treated in this course. Class notes will be available.) Handouts for the course will be available electronically as pdf files which are *best read on a computer with a reasonably large screen rather than a cell phone*.

You may want to look at:

Gura, Ein-Ya and M. Maschler, Insights into Game Theory, Cambridge U. Press, New York, 2008.

which originated as a doctoral thesis in mathematics education in Israel.

Other books you might want to look at are:

Straffin, Philip D. Game theory and strategy. Vol. 36. MAA, 1993. (MAA has issued an updated version of this book.)

DeVos, Matt and D. Kent, Game Theory: A Playful Introduction, American

Mathematical Society, Providence, 2016.

Assignments will be in the form of handouts and not from these books.

There will be problem sets to be handed in (several weeks after they are assigned), no midterm, but an in-class final (subject to pandemic issues) on the last scheduled day of class. Students taking the "doctoral" version of the course will also have to hand in a project towards the end of the semester. The project is optional for other students.

Readings and materials related to the course will be available at:

<https://www.york.cuny.edu/~malk/gametheory/index.html>

Please call to my attention any errors which you notice on the webpage above or on handouts.

On days that our class meets (Mondays) I will hold office hours prior to class (subject to change due to pandemic related issues).

Completed assignments and the project can be submitted as pdf files.