

Brief Introduction to Graph Theory (2022)

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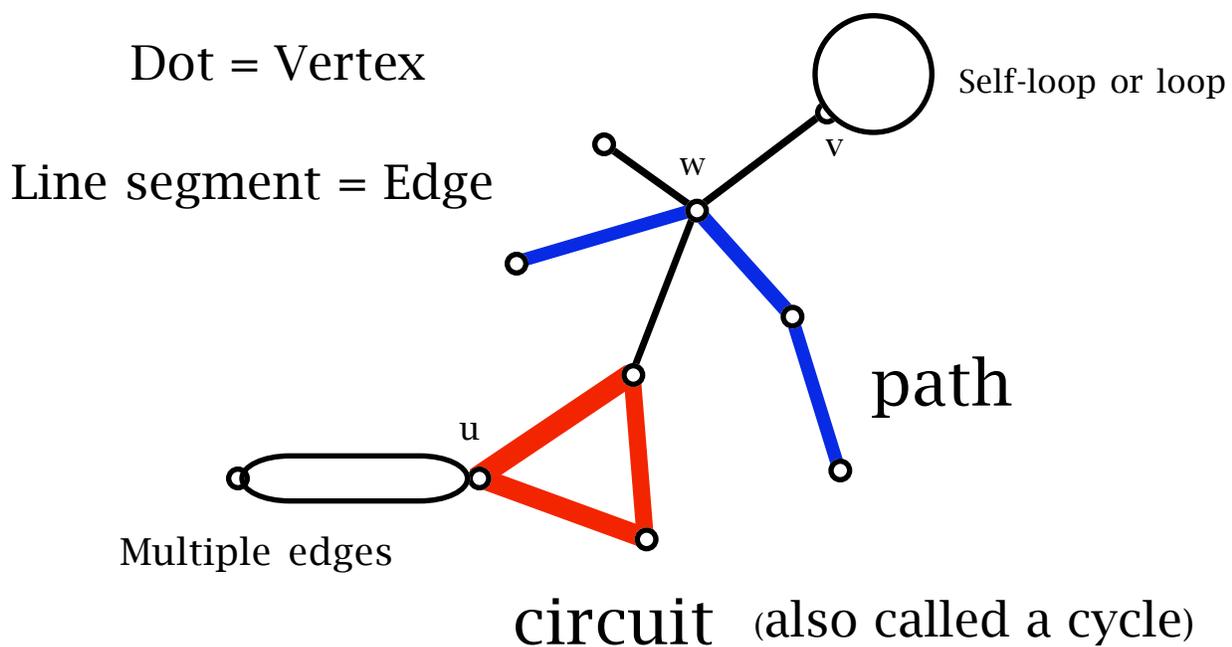
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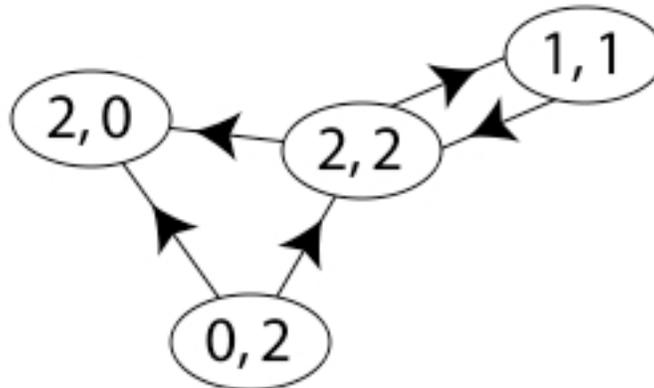
This graph has 10 vertices and 12 edges.

The valence or degree of a vertex in a graph is the number of (local) line segments which meet at the vertex. The valence of v is 3, of w is 5, and of u is 4.

The graph has three 1-valent vertices. Vertices of degree 1 are often called leaves by computer science scholars and biologists interested in graph

theory.

Graph theory provides a powerful tool for constructing mathematical models in a variety of situations. Given "objects" one can represent them by dots and the relationship between the objects can be indicated using straight or curved line segments joining these dots. Dots might represent people, street corners, or books. Line segments could indicate that the people are friends, that street corners are connected by a street, or that the books have the same author. In a graph edges don't have any direction but in a digraph (short for directed graph) one can use directed line segments (segments with arrows) to indicate, say, the direction that traffic can travel in down a street. The figure below (from Wikipedia) shows a digraph with 4 vertices and 5 directed edges. The numbers shown indicate in valence (indegree) and out valence (outdegree), first the number edges into the vertex, and then separated by a comma the number of edges out of the vertex.



Theorem: The sum of the indegrees equals the sum of the outdegrees and both of these quantities give the number of directed edges in the graph.

When a digraph represents the results of a round robin tournament (each pair of teams plays one game against each other team) where ties are not possible (every pair of teams is joined by a directed edge indicating which team beat the other) the indegree and outdegree of a vertex count the number games a team won and lost.

Theorem: (Basically due to Leonhard Euler - 1707-1783.)

In any graph the sum of the degrees adds to twice the number of edges in the graph.

Corollary: The number of vertices of odd valence (degree) in a graph is an even number.

Note: The number of edges or vertices of a graph can be even or odd.