

Example:

Consider the 2x2 matrix game below where Row and Column can each take two actions. A payoff like (x, y) means a payoff of x to Row from Row's point of view and a payoff of y to Column from Column's point of view. This is NOT a zero-sum game:

$(3, -2)$ $(-5, 2)$

$(4, -1)$ $(-4, -4)$

Nash's Theorem says that this game has pure strategy equilibria, mixed strategy equilibria or both. So if you are asked to examine this game for Nash equilibria you must do the following.

a. Use a motion diagram to look for pure-strategy equilibria. And there is one, at $(4,-1)$ from Row playing row 2 and Column playing column I.

b. Now WE MUST CHECK if there are any mixed strategy equilibria.

To do this, first write down

Row's game:

3 -5

4 -4

payoffs from Row's point of view and

Column's game:

-2 2

-1 -4

payoffs from Column's point of view.

To check if there is a mixed strategy equilibrium Row tries to design a spinner where Row 1 is played p percent of the time and Row 2 is played $1-p$ percent of the time, which will equalize Column's payoff from always playing column I or always playing column II.

So we see that $-2p - 1(1-p) = 2p - 4(1-p)$

would have to hold and there is a solution where $p = 3/7$. However, this does not mean there is a Nash mixed strategy equilibrium without checking what happens for Column.

Column wants to create a spinner which equalizes Row's payoffs from always playing Row 1 or always playing Row 2, leading to the equations below where q is the percentage of the time Column plays Column I and $(1-q)$ is the percentage of the time Column plays Column II.

Thus,

$$3q - 5(1-q) = 4q - 4(1-q)$$

So $0 = 1$ which means there is NO SOLUTION. (Had q turned out negative, there would also be no solution.) Hence, this game has NO MIXED STRATEGY Nash equilibrium. When we solved for p , if p turned out negative, again there would have been no mixed strategy Nash equilibrium.

So, for this example there is only one Nash equilibrium and it is a pure strategy equilibrium with payoff $(4, -1)$, so this game is not favorable from Column's point of view.