

Mathematical Insights into Electing America's President

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Some History:

2000

George W. Bush	50,456,062
Albert Gore, Jr.	50,996,582
Ralph Nader	2,858,843

Percentage for Bush/Gore: 47.9% 48.4%

Electoral college:

	Bush	Gore
Electoral vote	271	266
States carried	30	20 + DC

1992

William J. Clinton	44,909,326
George Bush	39,103,882
H. Ross Perot	19,742,240

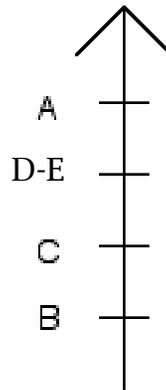
1968

Richard M. Nixon	31,785,148
Hubert H. Humphrey	31,274,503
George Wallace	9,906,473

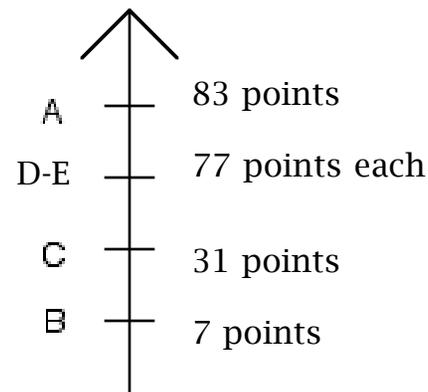
These elections show the danger of a "standard ballot" - vote for your favorite candidate without information about how the voters feel about other candidates.

Collecting more information from voters might be a good idea:

Here the voter is indifferent between D and E but likes A better than D or E but each of D and E better than C or B. This kind of ballot is called a preference or ordinal ballot.

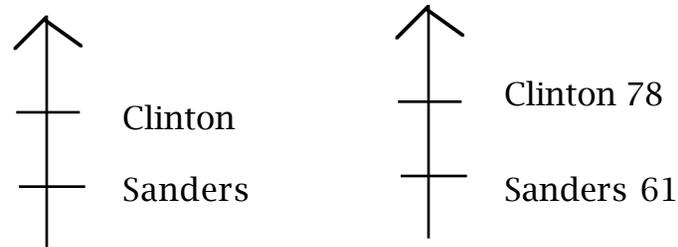


Example of one kind of cardinal ballot:



Each candidate can be assigned independently a number between 0 and 100 points.

Note that even in a two person vote in a primary, such as the April 19th vote in the Democratic Primary between Hilary Clinton and Bernard Sanders one might use "range" or "score ballots"



(each candidate could have been given between 0 and 100 points.)

Other ballots:

a. Vote only for those candidates you would be willing to have as President. (Approval voting)

b. Indicate your willingness to have each candidate be President by voting yes or no for each.

How reliable is the additional information voters provide with such more sophisticated ballots?

How does one compare the meaning of the the different number of points two different people give to a candidate?

From a mathematical modeling perspective elections involve:

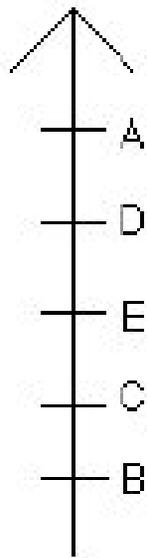
a. Voters

b. Choices

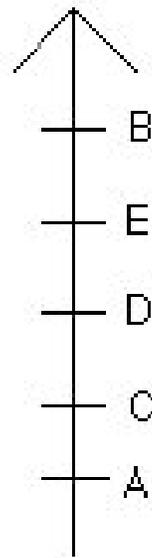
c. Ballots

d. Ways to use the ballots to obtain a "winner," "winners," or a ranking.

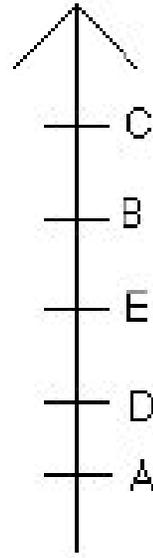
Ordinal ballot election:



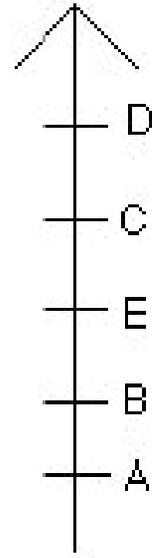
Votes: 18



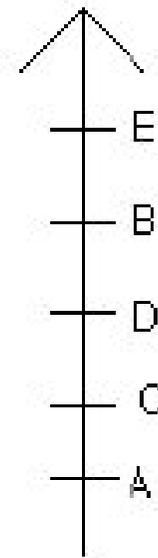
12



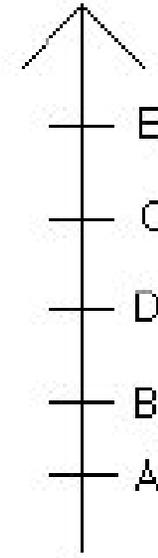
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9



4



2

Who deserves to be the winner of this election?

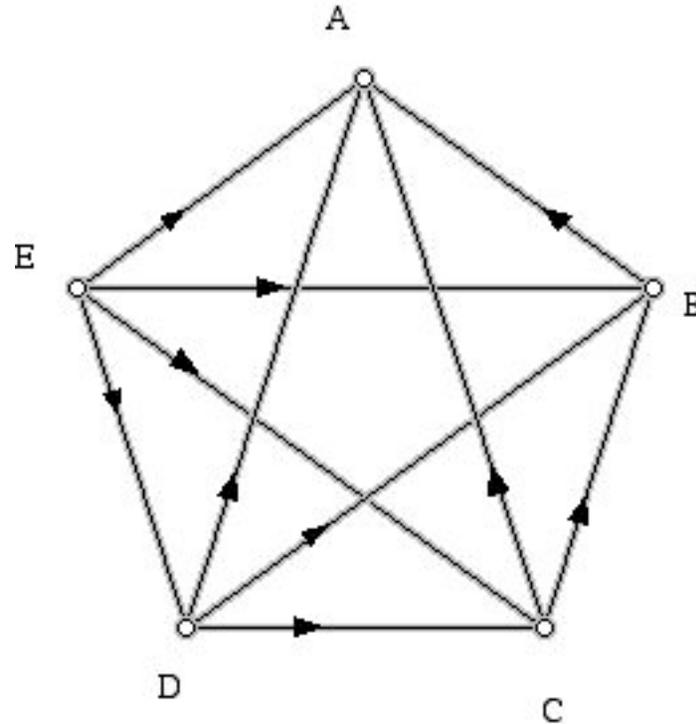
Whoever wins is usually viewed as the "people's choice" and expresses the "will" of the people.

American's tend to have faith that the "best" candidate wins in such important elections as those for President.

Using tradition as a guide, the way nearly all American elections are decided, A would be the winner!

Even though a majority of voters liked A the least and A's percentage of first place votes is only about 33 percent.

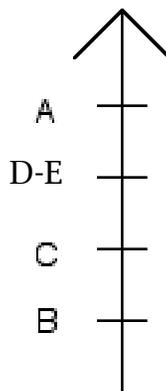
Here are the results of 2 way races: Perhaps you are surprised that E can beat each of the other candidates in a two-way race:



Perhaps the voting system should depend on "average" strength of the candidates with the voters.

Give points for how high up on a ballot the different candidates are.

This system is called the Borda Count, and the number of points for a candidate is the number of candidates below him/her.



A gets 4 points

B gets 0 points

C gets 1 point

D gets two point

E gets two points

If 10 voters with this ballot, multiply by 10.

Nifty and perhaps unexpected way to get Borda Count results when there is no tie.

Construct a matrix showing votes for the alternatives (candidates) in two-way races.

Here is the data about the two-way races

The entry in row i and column j is the number of votes for the row candidate in a race against the column candidate

	A	B	C	D	E
A	-	18	18	18	18
B	37	-	16	26	22
C	37	39	-	12	19
D	37	29	43	-	27
E	37	33	36	28	-

Remarkably: the row sums give a candidate's Borda Count:

$$\mathbf{B's\ Borda\ Count = (0)(18) + 4(12) + 3(10) + 1(9) + 3(2) + 1(2) = 101}$$

**It turns out that
D has the highest
Borda count so D
wins using this
election method.**

Here are some other methods sometimes used to decide elections using ordinal ballots:

Run-off

If no candidate gets majority that one eliminates all but the two candidates with the highest number of first place votes and conducts an election between them

In our example:

C, D, and E are eliminated and in the two-way race between A and B, B wins.

Sequential run-off

Instead of eliminating lots of candidates with not many first place votes all at once, one can eliminate them one at a time.

Round 1

A = 18; B = 12; C = 10; D = 9; E = 6 (Eliminate E)

Round 2

A = 18; B = 16; C = 12; D = 9 (Eliminate D)

A = 18; B = 16; C = 21 (Eliminate B)

A = 18; C = 37 C wins!

Presidential Election:

Direct election results:

a. ballot

b. decision method

Electoral college results:

a. weighted voting

b. weights are set by Huntington-Hill algorithm for the apportionment problem

To appear on the a state's ballot for election of the President a candidate must get the nomination of a party (the major ones are the Democratic and Republican parties) or get signed petitions from a large enough collection of voters.

The parties select nominees through a complicated process of primaries and caucuses.

However, when a primary occurs, as shown by the recent early Republican primaries, a candidate can "win" a state based on a small plurality of those who vote.

Arrow's Theorem

When there are 3 or more candidates no election method obeys a short list of fairness properties!

Kenneth Arrow majored in mathematics at City College!

Gibbard-Satterthwaite Theorem

When there are three or more candidates all election methods except dictatorship can be "manipulated!"

Gibbard is a philosopher and Satterthwaite a management scientist.

Weighted voting game:

[13; 9, 5, 5, 4, 2]

Minimal winning coalitions:

{1, 2} {1, 3} {1, 4} {2, 3, 4}

Player 5 has NO power!

Mathematics gives important insights into voting and elections situations

and more generally into many other questions which involve

fairness

* fair taxes

* bankruptcy

* fair division

*fair games

* fair legislative apportionment

Take home message:

Mathematics offers
many insights for
how to make our
democracy stronger!

Thanks for your
attention!