Refraction and Dispersion

1 Objectives

1. To understand refraction in optical systems, and
2. To understand dispersion in optical systems.

2 Introduction

From Einstein's Special Theory of Relativity, we learn that the maximum speed at which any object or information can travel is that of the speed of light in vacuum, usually denoted as c. Light traveling in any other medium travels at less than this speed. Because light must change speed when it crosses the boundary between materials, rays of light *refract*, or change direction. Further, the refractory properties of materials are different at different wavelengths; this frequency dependent behavior is known as *dispersion*. We will study both behaviors in this lab.

3 Theory

You learned in PHYS-151 that when a phenomenon is governed by the wave equation

$$\frac{\partial^2 A(x,t)}{\partial t^2} = v^2 \frac{\partial^2 A(x,t)}{\partial x^2} \ ,$$

the resulting wave trains are governed by a simple law

$$v = \lambda f \ ,$$

that is, the speed of wave propagation equals the product of the frequency and the wavelength. Einstein argued, and the data has repeatedly borne out the prediction, that the speed of light in a vacuum, $c = 299\,792\,458\,{}^{m}\!/\!{}_{s}$ exactly, is a universal constant for all observers. It is also a universal speed *limit*: nothing can travel faster than light in a vacuum. But that doesn't mean that light *always* travels at the speed c; in fact, in different materials, the propagation speed of light, v, is less (sometimes significantly!) than this. We define the *index of refraction* of a material as the ratio of c to v:

$$n = \frac{c}{v} \ ,$$

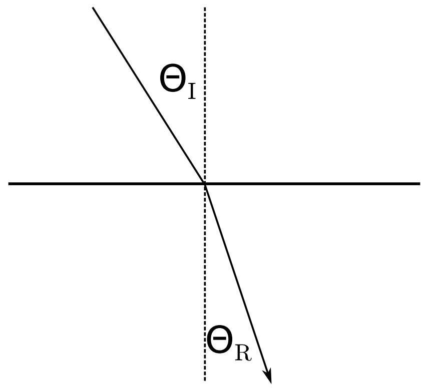


Figure 1: The definition of the incident and refracted angles. Light enters the interface at an angle θ_I with respect to the normal to the surface, and leaves the interface at an angle θ_R. If $\theta_I \neq \theta_R$, then the ray is said to be refracted.

and these values are always positive[1] and greater than 1. The refractive index of vacuum is exactly 1, while the refractive index of dry air at STP is so close to 1 (1.000 293) as to be indistinguishable in our labs. Your text lists the indices for a number of other materials, some with $n > 2$.

The frequency of the wave train is simply the number of cycles per unit time, and continuity requires that this value doesn't change at an interface between materials of different indices. As a consequence the *wavelength* of the wave must change across the boundary, and that can only happen if the wave changes direction! Because the different components of the reflected and refracted waves must add up to the total incoming wave, you can find a relationship between the angles of the incident and refracted rays (defined in Figure 1)

$$\frac{\sin\theta_I}{\sin\theta_R} = \frac{v_I}{v_R}.$$

Rewriting the v_i in terms of the n_i gives

$$n_I \sin\theta_I = n_R \sin\theta_R \ ,$$

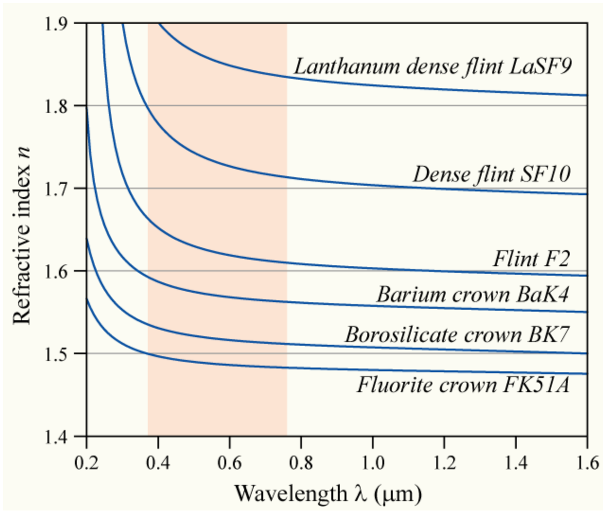
famously known as *Snell's Law* of refraction. If light slows down when crossing a boundary (a transition from low to high n), the rays bend towards the normal, and vice versa. You will use Snell's Law to determine the speed of light in different materials.

While the discussion above has repeatedly talked about *the* index of refraction of a material, it turns out there really is no such thing: the index of refraction depends on the frequency of the light! That is, we really have to replace relation $v = \lambda f$ with

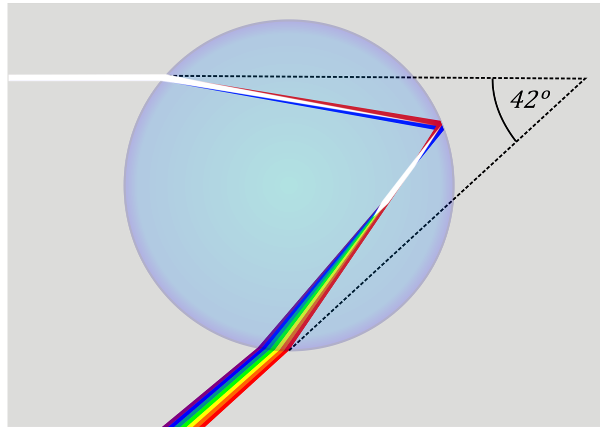
$$v(f) = \lambda f \ .$$

This more complicated relationship is known as a *dispersion relation*, and gives rise to frequency dependent refraction known as dispersion. While the difference from a constant speed

[1] You can, in fact, engineer *metamaterials* with negative indices in certain frequency bands; no such materials have ever been found in nature.



(a) Dispersion Curves



(b) Rainbows

Figure 2: The left-hand image shows the dispersion curves (wavelength vs index) for a number of different types of optical glass. The right-hand image show the process in a raindrop that creates a rainbow: a pair of dispersive refractions, with an intervening internal reflection.

is usually very small (often less than a percent across the optical frequencies, see Figure 2), it has a host of very well known consequences, the most interesting of which are the *chromatic aberration* of camera systems, and the formation of rainbows (again, see Figure 2). You will also explore dispersion in this lab.

4 Procedures

You will receive a number of clear blocks, laser pointers, protractors, white paper, and tape measures and meter sticks. You must use the information below to design and implement a series of experiments to measure refraction and dispersion in optical systems.

4.1 Index of Refraction

In the first two measurements, you'll be using Snell's Law to determine the index of refraction of two acrylic objects. Typically, these measurements in an undergraduate lab are taken with collimated white light sources, where dispersion and divergence conspire to complicate the measurements. Instead, we will use monochromatic, non-divergent light sources: diode based laser pointers; the rays consist of a single wavelength with a very small beam spread. You should be able to make excellent measurements with a modicum of care.

4.1.1 Acrylic Cube

A ray from a laser will travel across the lab in a straight line. If you intersect the beam with a slab with index of refraction n and parallel sides, the ray that emerges will be parallel to

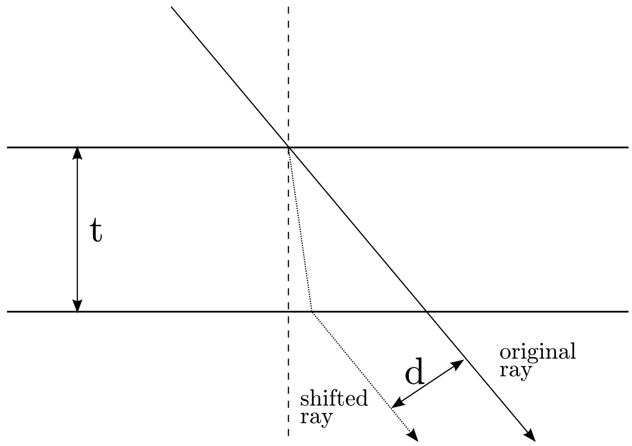


Figure 3: The offset distance, d, between the original ray and the refracted ray after passing through a slab of thickness t.

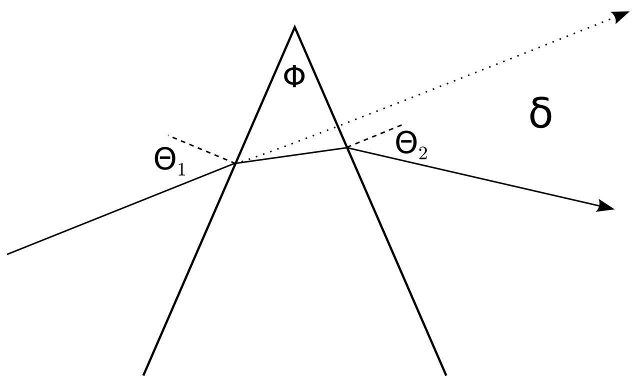


Figure 4: The angle of deviation, δ, for a ray passing through a prism. Light enters the interface at an angle θ_1 with respect to the normal to the surface, and leaves the interface at an angle θ_2. If $\theta_1 = \theta_2$, then δ is a minimum.

the original ray, but offset by a distance d; see Figure 3. By measuring incident angle θ_I and the offset distance d, you can extract the index of refraction.

Determine how to make this measurement. You must make a very careful measurement - the uncertainty of your result is very sensitive to the uncertainty of your angular measurements. While you have protractors available to you, and you should use them to sanity check your measurements, by themselves they won't give you good enough results.

4.1.2 Acrylic Prism

The *prism* is a triangular block of dispersive material, typically used to break a light source with multiple wavelengths into its constituent parts, much like a diffraction grating. They are also used in optical systems (such as cameras and periscopes) to transport or split images. Here, you'll use the prism to determine the index of refraction of the prism material.

Figure 4 shows how the path of light is deflected when passing through the prism. This deflection angle is called the *angle of deviation*. The minimum angle $\delta_{\min}$ occurs when the incoming and outgoing deflection angles are equal.[2] At this angle, the index of the prism material is given by

$$n = \frac{\sin\left(\frac{\Phi+\delta_{\min}}{2}\right)}{\sin\left(\frac{\Phi}{2}\right)} \ ,$$

assuming the prism is in air.

Measure the index of refraction for the prism. You can certainly use the minimum angle of deviation method. Additionally, there is a one angle measurement that gives the same information. What is that method?

4.2 Dispersion

Acrylic polymers are well matched to optical transmission, because their dispersion curves are relatively constant across optical wavelengths. Still, with sufficient care, observation of dispersion is not difficult. Here, you will receive three different colored laser pointers and a number of acrylic blocks; devise an experiment to prove that these different frequencies travel at different speeds through the acrylic.

[2] You could consider proving this...

Pre-Lab Exercises

Answer these questions as instructed on Blackboard; make sure to submit them before your lab session!

1. A ray hits an object at 15° from the vertical. If the material has index of refraction 1.5, what is the refraction angle?
2. If a ray is incident on a block of material at 15° from the vertical, and is refracted to 8° from the vertical, what is the index of refraction?
3. A ray of light crosses from a flint glass slab at 20° from the normal into air. At what angle does it emerge?
4. A ray of light passes through a prism, making the minimum angle of deviation. The rays enter and emerge at 7° from the normal to the surface. What is the index of refraction of the material, if the prism angle is 50°?

Post-Lab Exercises

1. What measurement did you devise in Section 4.1.1? What is the index of refraction for you laser in the acrylic? What does this give as the speed of light in the acrylic? Estimate the uncertainty of your result.

2. What measurement did you devise in Section 4.1.2? What is the index of refraction for you laser in the acrylic? What does this give as the speed of light in the acrylic? Estimate the uncertainty of your result.

3. What experiment did you devise in Section 4.2? How does your data prove that different wavelengths travel at different speeds through the material? What are the corresponding speeds of light in the acrylic? Estimate the uncertainty of your result.

4. Discuss briefly whether you have met the objectives of the lab exercises.